# The Proposed Method for Determining Stiffness of Shear Walls Weakened by Openings in Buildings with Load-Bearing Wall Structures

Jasiński Radosław

Laboratory of Civil Engineering Faculty and Department of Building Structures, Silesian University of Technology, Akademicka 5, 44-100 Gliwice, Poland, e-mail: Radoslaw.Jasinski@polsl.pl; ORCID: https://orcid.org/0000-0003-4015-4971

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*Abstract*

*Openings in masonry shear walls can cause many problems during load distribution. Forces in walls are distributed proportionally to stiffness including a potential rotation of the building, which is determined by the RC point. Stiffness of wall units can be determined with traditional methods used in the mechanics of constructions and the theory of elasticity. However, in the presence of openings the wall should be considered as material susceptible to tensile forces. The fundamental objective of this paper is to describe issues concerning the design process of masonry shear walls in buildings with conventional load-bearing wall structures and to demonstrate the procedure for determining stiffness of wall units with openings. Two methods for determining stiffness are proposed in this paper. Method I presents the relationships to determine stiffness of a wall unit regardless of the configuration of openings in the wall units. Method II describes the simplified relationships to determine stiffness of a wall unit with openings arranged in rows. The presented methods can be also put in practice to distribute load in simple-shape buildings.*

**Keywords:** *Masonry Structures, Masonry Stiffening Walls, Walls with Openings, Stiffness of Wall*

## 1. Introduction

The proper arrangement of supporting walls which take external horizontal loads should provide the required spatial stiffness of a building. The construction system should be designed in such a way to obtain the symmetrical arrangement of shear walls. Then, the load distribution on individual walls in a specific direction is similar, and at the same time spatial stiffness of the building is the highest. However, in many cases when the building plan is irregular, there are one-way floors or the arrangement of supporting walls is not symmetrical (due to functional properties), horizontal loads can result in torsion of a building and an increased displacement of walls. The standards [1] and [2] clearly specify that if coordinates of the point, at which applied external loads (LC) differ from coordinates of the torsion centre (RC) (specified in EC-6 as the stiffness centre) of the plan of stiffening walls, then the torsional effect should be taken into account. No detailed information has been specified regarding the method of determining the torsion centre of a building or a way to include the torsional effect while providing the freedom of the calculation method.

External forces in stiffening systems and walls, and in structural parts (wall strips and connectors – lintels and spandrel areas) can be determined with various calculation methods that differ in labour-consumption and precision in determining internal forces against numerical calculations. Generally, the spatial analysis of the integrated stiffening structure or simplified methods, which include division into separated stiffening systems, can be applied.

To simplify the calculations, Eurocode assumes that loads can be only taken by walls parallel to external loading, and the load distribution proportional to wall stiffness can only take place in case of rigid floors. Based on this assumption, the integrated spatial structure can be divided into separate stiffening systems or even wall strips with interacting fragments of transverse walls. The fundamental objective of this paper is to describe issues concerning the design process of masonry shear walls in buildings with conventional load-bearing wall structures and to demonstrate the procedure for determining stiffness of wall systems with openings. This paper is a continuation of previous papers by the author on analyses of shear walls [3, 4, 5]. This paper describes general information and original methods of determining stiffness of masonry shear walls with opening of any shape and configuration.

## 2. General information

The place of torsion centre (RC) of the wall plan is determined by dividing the stiffening structure into stiffening systems in the direction of the building length and width. In accordance with general rules suggested by Eurocode, stiffness values $K_{x,i}$ and $K_{y,j}$ are calculated and distance $a_{x,i}$, $a_{y,i}$ is determined between torsion centres and the point, at which external loads (LC) are applied. Coordinates of the torsion centre (RC) of walls are determined from the condition of force equilibrium against the point (LC) and calculated from the following equations:

$$a_x = \frac{\sum_i \left(a_{x,i} K_{x,i}\right)}{\sum_i K_{x,i}}, \quad a_y = \frac{\sum_i \left(a_{y,i} K_{y,i}\right)}{\sum_i K_{y,i}}, \tag{1}$$

where:

$a_{xi}$, $a_{yi}$ – distance between the point of applied external load LC and torsion centres of individual stiffening systems or walls.

Coordinates of the centre RC position are used to estimate the correctness of the analysed stiffening system for the building. In the systems with symmetrically arranged walls, there is no eccentricity of loading with reference to the torsion centre of a building, which can overlap the point of applied load. For asymmetrically arranged walls, the horizontal force acting at eccentricity causes a significant torsion of the building. However, in the building with irregularly arranged walls, the obtained torsional moment is not

compensated by shear forces generated in the walls.

When the torsion centre of the building is known, internal forces in individual stiffening systems, and then in structural parts, can be determined. While choosing the stiffening systems, any wall fragments or strips which do not determine the spatial stiffness of the building, can be neglected. Thus, the following elements can be excluded: slender piers between windows, non-structural infill walls that do not interact with the infilled structure, fragments of walls connected with stiffening walls by slender connectors, or single masonry columns.

The method of overall stiffness of the stiffening system (method I) or the simplified method of overall stiffness (method II) can be used to determine coordinates of the torsional centre of a building, and then to determine internal forces in structural elements.

## 3. Methods for determining stiffness

### *3.1. Method I – overall stiffness of the stiffening system*

The method recommended by American and Canadian standards [6, 7], consisting in separation of independently operating stiffening systems from the structure, was used to distribute load into stiffening systems. Openings in walls do not have to regularly arranged (by separating vertical strips, but can differ in shape and size.

The paper [8] proposed to determine stiffness of the wall with openings by dividing it into zones weakened by openings and zones of increased stiffness (lintels and spandrel areas) – Fig. 1. Determination of overall horizontal displacement from the unit load imposed on the top edge of the wall with the concentrated force is required to determine stiffness of the whole stiffening system. The total displacement of the top edge of the wall is a sum of displacements of spandrel areas, pillars between openings, and lintels, and is determined from the following relationship:

$$\Delta_w = {}^{A}\Delta_w + {}^{P}\Delta_w + {}^{B}\Delta_w, \tag{2}$$

where:

${}^{A}\Delta_w$, ${}^{B}\Delta_w$ – displacement of the spandrel area and the lintel, ${}^{P}\Delta_w$ – displacement of a fragment of wall with openings having a height $h_0$ and a length $l$.

Displacements of wall components are determined from the relationship between geometry and boundary conditions for the wall. If the ratio of the wall height to its length is $h/l > 2$ (slender elements), then the effect of shear stresses can be neglected while determining the wall stiffness. Otherwise, the wall stiffness should be determined with the included shear strains (thick walls). Stiffness values of the wall elements fixed in a different way and at different size ratio, whose ends are loaded with the concentrated force, are shown in Table 1 from [9, 10].

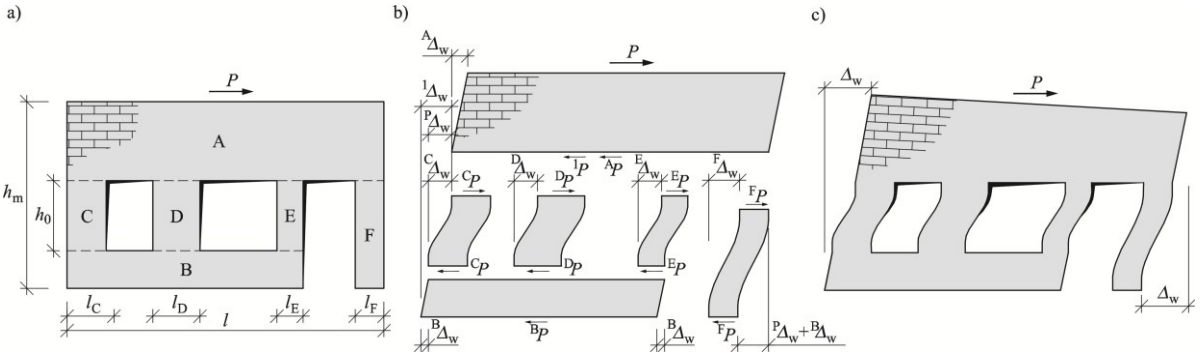
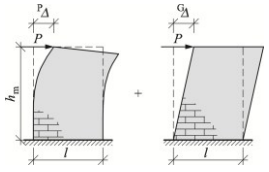
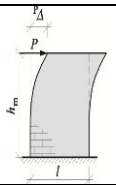
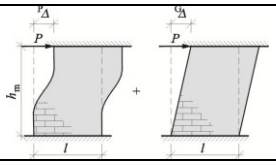
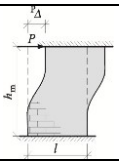



Fig. 1 The method of total stiffness of the stiffening unit: a) division of the wall with openings into component elements, b) horizontal deformations of the components of the wall, c) total deformation of the wall caused by horizontal loads

Tab. 1 Stiffness of walls subjected to horizontal loading [9, 10]

| Wall scheme | $h/l \leq 2$ | $h/l \geq 2$ |
|---|---|---|
| Cantilever wall "C" | $K_w = \dfrac{1}{\dfrac{h_m^3}{3EI} + \dfrac{1{,}2h_m}{GA}}$ | $K_w = \dfrac{3EI}{h_m^3}$ |
| |  |  |
| Wall fixed at both ends "F" | $K_w = \dfrac{1}{\dfrac{h_m^3}{12EI} + \dfrac{1{,}2h_m}{GA}}$ | $K_w = \dfrac{12EI}{h_m^3}$ |
| |  |  |

Displacements of structural elements of the walls are determined from equations presented in Table 1 assuming the load to be $P$=1. After determining the total displacement of the wall, its stiffness is calculated from the following relationship:

$$K_w = \frac{1}{\Delta_w}, \tag{3}$$

where:

$\Delta_w$ – total displacement of the top edge of the wall under the unit load $P$=1.

Stiffness of the wall shown in Fig. 1 can be calculated by dividing it into structural elements. Criteria for force equilibrium in the cross-section below the lintel area can be expressed as:

$$\begin{gathered} P = \underbrace{{}^{C}P + {}^{D}P + {}^{E}P}_{{}^{1}P} + {}^{F}P = {}^{F}\Delta_w K_z + {}^{F}\Delta_w K_F \\ {}^{1}\Delta_w = {}^{F}\Delta_w = \frac{P}{K_z + K_F} \end{gathered}, \tag{4}$$

where:

$K_z$ – stiffness of the system consisting of stiffness of the spandrel area and pillars between window openings.

Stiffness $K_z$ is determined from the criterion for a sum of displacements in the cross-section below the lintel area:

$$\begin{gathered} {}^{1}\Delta_w = {}^{B}\Delta_w + {}^{P}\Delta_w = \frac{{}^{1}P}{K_B} + \frac{{}^{1}P}{\sum {}^{P}K_w}, \\ \frac{{}^{1}P}{K_z} = \frac{{}^{1}P}{K_B} + \frac{{}^{1}P}{\sum {}^{P}K_w} \rightarrow \frac{1}{K_z} = \frac{\sum {}^{P}K_w + K_B}{K_B \sum {}^{P}K_w} \rightarrow K_z = \frac{K_B \sum {}^{P}K_w}{\sum {}^{P}K_w + K_B}. \end{gathered} \tag{5}$$

$${}^{A}\Delta_w = \frac{P}{K_A}$$

Taking into account relationship (4) and (5) and , the overall horizontal displacement of the top edge of the wall according to (2) is determined from the following equation:

$$\begin{gathered} \Delta_w = {}^{A}\Delta_w + \underbrace{{}^{P}\Delta_w + {}^{B}\Delta_w}_{{}^{1}\Delta_w = {}^{F}\Delta_w}, \\ P\frac{1}{K_w} = P\frac{1}{K_A} + P\frac{1}{K_z + K_F} \rightarrow \frac{1}{K_w} = \frac{1}{K_A} + \frac{1}{K_z + K_F} = \frac{K_z + K_F + K_A}{K_A\left(K_z + K_F\right)} \\ \frac{1}{K_w} = \frac{K_z + K_F + K_A}{K_A\left(K_z + K_F\right)}. \end{gathered} \tag{6}$$

Finally, stiffness of the wall shown in Fig. 1 is expressed as:

$$K_w = \frac{K_A\left(K_z + K_F\right)}{K_z + K_F + K_A}, \tag{7}$$

where:

$\sum {}^{P}K_w = K_C + K_D + K_E$ – a sum of stiffness of pillars between openings,

$K_z = \frac{K_B \sum {}^{P}K_w}{\sum {}^{P}K_w + K_B}$.

Such an approach can be totally acceptable providing that an increase in a height of pillars between openings caused by cracks in tension opening corners is neglected. This process can be observed at very low shear loads [3, 4] which appear much earlier than cracks in internal or edge fragments of the wall. The research [11] showed that such a process could be considered as acceptable regardless of the value of initial compressive stress [11]. Cracks in corners of the openings increased a height of pillars between these openings. At relatively low values of compressive stress, an inclined crack can reach the ring beam in the top zone of the opening, or two or three layers of masonry units below the spandrel area. Horizontal load acting from the opposite direction produces identical cracks in opposite corners of the openings. As a result, the crack will be symmetrical. Therefore, it is reasonable to determine stiffness of pillars between openings taking into account a height increased by the range of cracks. This approach clearly reduces stiffness of walls with opening when compared to determined stiffness with the neglected effect of cracks in corners of the openings. The greater the range of cracks is, the greater the height of a pillar and the lower stiffness are. In the majority of design cases, a range of cracking can be assumed to cover one layer of masonry units regardless of the presence of absence of reinforcement in bed joints. Only circumferential reinforcement in openings can reduce the development of cracks, and consequently can reduce a height of pillars.

Stiffness of walls with different shapes and types of openings can be determined in a similar way. For an individual stiffening wall with a door opening – Fig. 2, the wall stiffness is determined from the overall displacement of the wall and the criterion for force equilibrium in the top edge of pillars between openings (4) according to the following relationships:

$$\Delta_w = {}^{A}\Delta_w + {}^{P}\Delta_w \rightarrow P\frac{1}{K_w} = P\frac{1}{K_A} + P\frac{1}{\sum {}^{P}K_w}, \tag{8}$$

$$\frac{1}{K_w}=\frac{1}{K_A}+\frac{1}{\sum^P K_w}=\frac{\sum^P K_w}{K_A\sum^P K_w}+\frac{K_A}{K_A\sum^P K_w}=\frac{1}{K_W}=\frac{K_A+\sum^P K_w}{K_A\sum^P K_w},$$

$$K_W=\frac{K_A\sum^P K_w}{K_A+\sum^P K_w},$$

where:
$K_A$, $K_B$ – stiffness of lintels and spandrel areas reduced by a height of the masonry unit,
$\sum^P K_w = K_C + K_D$ – a sum of stiffness of pillars between door openings with a height increased by a height of the masonry unit $h_u$.

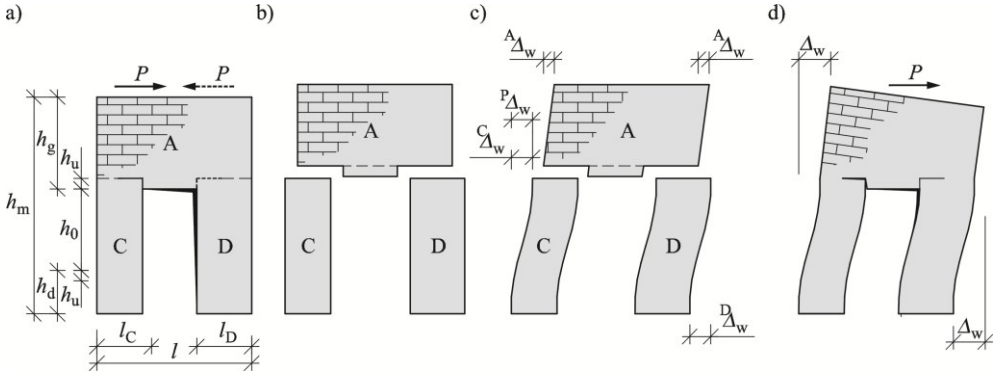



Fig. 2 The method of total stiffness of the wall stiffening unit with a single door opening: a), b) division of the wall with openings into component elements, c) horizontal deformations of the components of the wall, d) total deformation of the wall caused by horizontal loads

Following the similar procedure, stiffness of an individual stiffening wall with a single window opening – Fig. 3 – is expressed by the following equation:

$$\Delta_w={}^A\Delta_w+{}^P\Delta_w+{}^B\Delta_w \rightarrow P\frac{1}{K_w}=P\frac{1}{K_A}+P\frac{1}{\sum^P K_w}+P\frac{1}{K_B},$$

$$\frac{1}{K_w}=\frac{1}{K_A}+\frac{1}{\sum^P K_w}+\frac{1}{K_B}=\frac{K_B\sum^P K_w+K_AK_B+K_A\sum^P K_w}{K_AK_B\sum^P K_w}, \quad (9)$$

$$K_w=\frac{K_AK_B\sum^P K_w}{\sum^P K_w(K_B+K_A)+K_AK_B}.$$

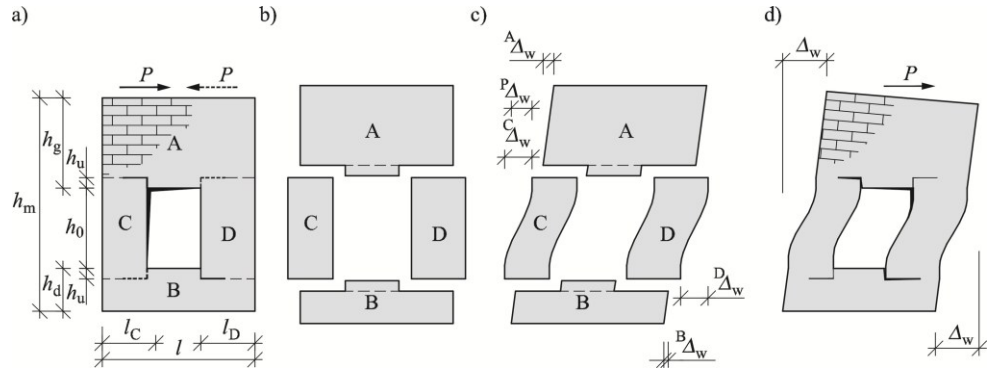



Fig. 3 The method of total stiffness of the wall stiffening unit with a single window opening: a), b) division of the wall with openings into component elements, c) horizontal deformations of the components of the wall, d) total deformation of the wall caused by horizontal loads

For an individual stiffening wall with a single door opening and a single window opening– Fig. 4 – the wall stiffness is expressed by the following equation:

$$\Delta_w={}^A\Delta_w+\underbrace{{}^P\Delta_w+{}^B\Delta_w}_{{}^1\Delta_w={}^E\Delta_w},$$

$$P\frac{1}{K_w}=P\frac{1}{K_A}+P\frac{1}{K_z+K_E}\rightarrow\frac{1}{K_w}=\frac{1}{K_A}+\frac{1}{K_z+K_E}=\frac{K_z+K_E+K_A}{K_A(K_z+K_E)}, \quad (10)$$

$$K_w=\frac{K_A(K_z+K_E)}{K_z+K_E+K_A},$$

where:

$\sum {}^{P}K_w = K_C + K_D$ – a sum of stiffness of pillars between window openings,

$K_z = \dfrac{K_B \sum {}^{P}K_w}{\sum {}^{P}K_w + K_B}$ – stiffness of the system consisting of the spandrel area and pillars between window openings.

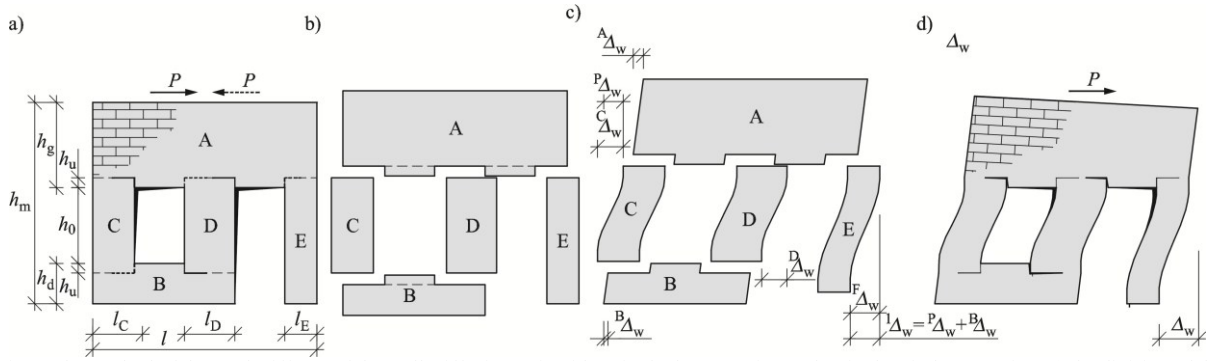



Fig. 4 The method of the total stiffness of the wall stiffening unit with a single door opening and a single window opening: a), b) division of the wall with openings into component elements, c) horizontal deformations of the components of the wall, d) total deformation of the wall caused by horizontal loads

When stiffness is determined, then values of shear forces acting on individual structural elements of the wall can be calculated from the following relationships (7) – (10). At first, the overall displacement $\Delta_w = P/K_w$ is calculated, and then displacements of individual structural elements ${}^{A}\Delta_w$, ${}^{B}\Delta_w$, ${}^{P}\Delta_w$ are determined.

### *3.2. Method II – simplified method of overall stiffness of the stiffening system*

The proposed method refers to the method of separated stiffening systems employed in Poland [12, 13, 14] and Europe [15, 16], and the method of overall stiffness. The proposed method is not applicable to walls, in which openings are arranged in a regular mode and do not form strips running throughout the building height. The procedure for determining stiffness of individual walls is less labour intensive than the method I (for overall stiffness) as the stiffening system does not require division into structural elements.

Stiffness of the walls $K_w$ in the direction of the building length and width is determined neglecting the presence of openings, but including fragments of perpendicular walls which interact with walls of the stiffening system. The distribution of loads requires the reduction of the stiffening system by using the stiffness reduction factor $\eta$, whose value is determined by the stiffness of lintels and the number of opening stripes. The reduced stiffness of the stiffening system is determined from the following relationship:

$$B_w = \frac{E_i J_{i,j}}{\eta}, \tag{11}$$

where:

$E_i J_{i,j}$ – determined stiffness of the wall with neglected openings and included sections of interacting walls perpendicular to the stiffening wall,

$\eta$ – stiffness reduction factor of the stiffening system including stiffness of lintels and the number of openings, which depends on the ratio between lintels height and span.

The parameter $\eta$ depends on the height $h_n$ to the clear span of a lintel $l_n$ ratio and the number of opening rows. In case of walls with door openings, where only lintels are present, a height $h_n$ is the lintel height, and in case of walls with window openings $h_n$ of lintels is calculated from the following equation:

$$h_n = \sqrt[3]{h_{ng}^3 + h_{nd}^3}, \tag{12}$$

where:

$h_{ng}$ – a height of lintel,

$h_{nd}$ – a height of spandrel area.

The estimated length of the stiffness reduction factor $\eta$ (recommended by the standard BN-8812-01:1974 [17] for calculations made for reinforced concrete buildings) depends on the number of opening rows and the ratio between the lintel height and span $h_n / l_n$ and can be directly read from the graph shown in Fig. 5 or Table 2.

Tab. 2. Values of coefficient $\eta_i$ specified in [17]

| Ratio of lintel height and span $h_n / l_n$ | Number of opening rows | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | ≥ 4 |
| $0.2 \le h_n / l_n \le 0.4$ | 2.2 | 2.7 | 4.0 | 6.0 |
| $0.4 \le h_n / l_n \le 0.7$ | 1.5 | 1.7 | 2.1 | 2.8 |
| $h_n / l_n > 0.7$ | 1.1 | 1.3 | 1.5 | 1.8 |

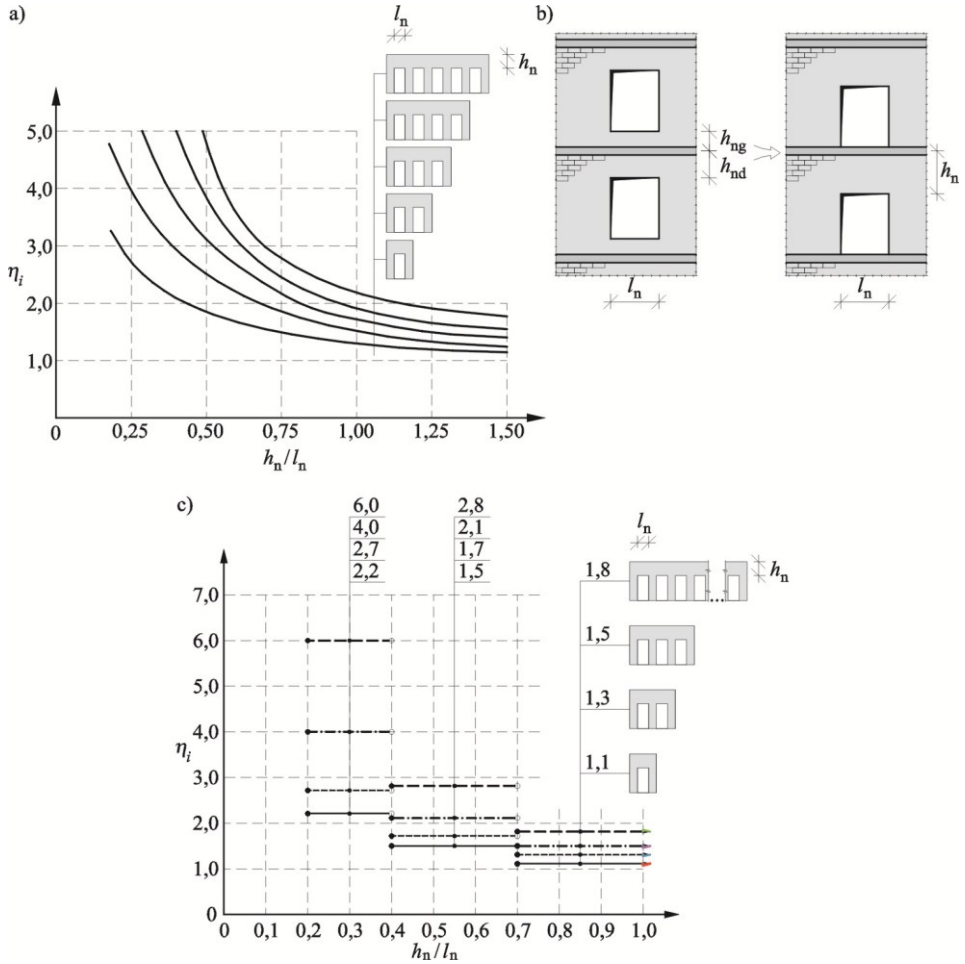



Fig. 5 The principle of determining the $\eta_i$ coefficient used for calculations of reinforced concrete and masonry structures [18, 19, 13]: a) the $\eta_i$–$h_n/l_n$ relationships, b) the principle of determining the substitute lintel height $h_n$, c) the values of the $\eta_i$ coefficients adopted in the standard [17]

## 4. Conclusion

Openings in masonry shear walls can cause many problems during load distribution. Values of forces in walls depend on both loads acting on the building and the position of the RC point, whose coordinates are determined by the wall stiffness. Stiffness of wall units can be determined with traditional methods used in the mechanics of constructions and the theory of elasticity. However, in the presence of openings the wall should be considered as material susceptible to tensile forces. This paper presents two methods for determining the stiffness of walls weakened by openings. Method I presents the relationships to determine stiffness of a wall unit regardless of the configuration of openings in the wall units. Method II describes the simplified relationships to determine stiffness of a wall unit with openings arranged in rows. The presented methods can be also put in practice to distribute load in simple-shape buildings.

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