Study of Thermal Buckling in Thin Eco-Concrete Panels Reinforced with Cow Bone Particulates

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ABSTRACT

The pursuit of sustainable construction has become a central objective in modern civil engineering, driven by the urgent need to minimize environmental impact. Integrating recycled waste into construction materials represents a cornerstone of sustainability. It not only mitigates ecological burdens but also transforms waste into valuable resources for construction industry. Cow bones, which are commonly generated as waste from households, slaughterhouses, and meat markets, represent a promising alternative material due to their availability, composition, mechanical strength and slow degradation. The aim of this study is to assess whether cow bone powder can enhance the thermal stability of concrete plates while partially substituting conventional materials such as cement or fine sand. Specifically, it examines the thermal buckling behavior of a thin eco-concrete panel reinforced with cow bone particulates. After being crushed and homogenized, the cow bone particles are incorporated into the concrete mixture to evaluate their influence on the critical temperature at which thermal buckling occurs. The effective thermo-mechanical properties are estimated using the FU–SHY homogenization rule. Classical Plate Theory, First-Order Shear Deformation Theory, and Higher-Order Shear Deformation Theory are then used to predict the critical buckling temperature. The findings indicate that increasing the cow bone powder content reduces stiffness, density, and thermal conductivity, while slightly increasing Poisson's ratio and thermal expansion. However, the addition of cow bone powder adversely affects thermal stability by lowering the critical buckling temperature.

KEYWORDS: e*co-concrete; recycled waste; cow bone powder; thermal buckling; sustainable materials; concrete panels*

1 INTRODUCTION

Concrete is one of the most widely used materials in civil engineering because of its high compressive strength, durability, availability, and adaptability to structural applications. However, conventional concrete production causes important environmental impacts, mainly due to Portland cement consumption, natural aggregate extraction, and CO_2 emissions.

Consequently, many researchers have investigated recycled, agricultural, industrial, and bio-sourced wastes as alternative materials for sustainable concrete.

Adekanmi [1] studied the use of cow dung powder in blended concrete and showed that this agricultural waste can affect compressive strength and contribute to the development of eco-friendly cementitious materials. Adewole and Oladele [2] investigated cow bone ash-reinforced polyester composites and demonstrated that the particle size distribution of cow bone ash strongly influences mechanical behavior, confirming the reinforcing potential of bone-based particles. Adetayo et al. [3] evaluated pulverized cow bone ash and waste glass powder for improving the geotechnical properties of tropical laterite, showing that these wastes can enhance soil performance. Adetayo et al. [4] also studied ternary blends of pulverized cow bone ash and waste glass powder in concrete and reported that these materials can improve strength while reducing Portland cement consumption.

The American Concrete Institute [5] provides design recommendations and mechanical property references useful for comparing the performance of cementitious and reinforced materials. Ararsa [6] investigated Ambo sandstone fine aggregate as an alternative river sand replacement and showed the importance of local aggregate substitution in normal concrete production. The BA 60 rules [7] provide early reinforced concrete calculation regulations and are useful as a reference framework for structural concrete design. The BAEL 80 rules [8] introduced reinforced concrete design recommendations used in structural engineering practice. The BAEL 91/99 rules [9] provide revised reinforced concrete regulations and are commonly used for comparing calculated concrete properties with standard design values.

Belmahi et al. [10] studied the influence of acidic and alkaline environments on cement mortars and showed that aggressive environments significantly affect mechanical performance. Bencharif et al. [11] developed a hybrid BEM-TLM-PML method for calculating dynamic impedance functions of rigid strip foundations on poroelastic soil, contributing to soil–structure interaction modeling. Bencharif et al. [12] also presented a time-domain implementation of soil–structure interaction techniques with frequency-dependent impedance functions. Benfrid et al. [13] analyzed the thermo-mechanical behavior of glass powder-based eco-concrete panels and showed that waste glass powder can influence both thermal and mechanical performance. Benfrid et al. [14] further studied the selective integration of waste-derived glass nano-powders in structural wall concrete and demonstrated improvements in thermal efficiency and elasto-mechanical properties.

British Standards [15] provide structural design recommendations and are useful for validating concrete and masonry-related mechanical properties. Cha et al. [16] investigated the adsorption of lead ions onto waste cow bone powder surfaces and demonstrated the environmental potential of cow bone powder for removing Pb^{2+} from contaminated water. Chatbi et al. [17] analyzed nano-clay platelet integration in concrete beams resting on elastic foundations and showed its influence on bending performance. Chatbi et al. [18] investigated free vibration of composite beams reinforced with randomly aligned carbon nanotubes resting on elastic foundations, contributing to dynamic analysis of reinforced composites. Chatbi et al. [19] studied nano-SiO_2 reinforced concrete raft foundations resting on elastic foundations and showed that homogenized properties are essential for predicting bending behavior. Chatbi et al. [20] also developed computational models for vibration and modal analysis of silica nanoparticle-reinforced concrete slabs with elastic and viscoelastic foundation effects.

Cheng [21] analyzed thermoelastic problems of concrete rectangular thin plates and provided reference results that are useful for validating thermal buckling models of concrete panels. Daarol et al. [22] investigated pulverized cow bone as a partial cement replacement in foam concrete and observed that moderate replacement levels can improve compressive strength. Dagnaw et al. [23] studied partial replacement of cement with animal bone ash in

concrete at elevated temperatures and showed its relevance under thermal exposure. Edun et al. [24] recycled animal bone as a partial replacement for coarse aggregate in lightweight hollow sandcrete blocks, demonstrating its potential in lightweight construction products. Elhennani et al. [25] examined buckling and free vibration of nanoparticle-reinforced concrete beams resting on multi-parameter elastic foundations.

Eurocode 2 [26] provides design rules for concrete structures and is used as an important reference for comparing effective concrete properties with standard design requirements. Fu et al. [27] proposed correction methods for measuring fiber length in short-fiber-reinforced thermoplastics, providing a useful theoretical basis for homogenization and effective property estimation. Harrat et al. [28] modeled the dynamic response of clay nanoparticle-modified concrete beams resting on two-parameter elastic foundations. Harrat et al. [29] studied the static behavior of nano-SiO_2 based concrete beams on elastic foundations and demonstrated the positive effect of nanoparticles on structural response. Harrat et al. [30] analyzed nano-Fe_2O_3 reinforced concrete slabs exposed to temperature fields and supported by viscoelastic foundations.

Hawaz et al. [31] examined animal bone powder as a partial replacement for cement in normal concrete and showed that concrete performance depends strongly on the replacement dosage. Ikumapayi et al. [32] prepared and characterized cow bone powder using SEM, EDX, XRF, image segmentation, and particle size analysis, confirming its suitability for composite applications. Kecir et al. [33] investigated nano-sized iron oxide particles in concrete raft foundations using non-local bending analysis and showed their influence on mechanical performance. Khan et al. [34] studied waste glass powder as a partial cement replacement and confirmed its potential for sustainable concrete production. Lu [35] analyzed the influence of recycled mortar powder on concrete performance and showed that fine recycled powders affect concrete behavior.

Ogarekpe [36] studied burnt and crushed cow bones as partial replacement for fine aggregate in concrete and reported effects on water demand, setting behavior, and compressive strength. Omah et al. [37] characterized the thermo-physical performance of bone particles proposed for polymer composite production and confirmed the material potential of bone-based particles. Palomino-Guzmán [38] proposed beef and pig bone waste as cement replacement for sustainable concrete production, highlighting the valorization of animal bone waste. Punyanitya et al. [39] studied retrograded rice starch and cow bone powder composite sponge and showed that cow bone powder can be used in bio-composite materials. Quadri et al. [40] investigated cow bone powder as a partial substitute in sandcrete blocks and showed that its mineral composition makes it suitable for cementitious applications. Quadri and Wasiu [41] assessed concrete with partial cement replacement by cow bone powder and confirmed its influence on mechanical properties.

Rasulov et al. [42] studied the production of NP fertilizers based on phosphate decomposition, which is relevant because bone waste contains phosphate-rich compounds and can be valorized chemically. Reddy [43] developed advanced theories for elastic plates and shells, including shear deformation effects, and this work provides the theoretical foundation for plate modeling. Sari et al. [44] transformed cow bone waste into phosphate fertilizer and showed the chemical value of cow bone due to its phosphate content. Sayyad et al. [45] studied thermal buckling of simply supported moderately thick functionally graded plates and demonstrated the importance of shear deformation theories in predicting critical temperature. Sruthy [46] investigated concrete made with cow dung ash and glass fiber, confirming the usefulness of organic waste in cementitious composites. Timoshenko and Gere [47] established the classical theory of elastic stability, which remains a fundamental basis for buckling analysis. Zahafi et al. [48] developed a lumped parameter model for

vertical vibrations of surface circular foundations on nonhomogeneous soil, contributing to structural and foundation vibration analysis.

Despite these contributions, most existing studies on cow bone powder, animal bone ash, recycled powders, and particle-reinforced concrete have mainly focused on material characterization, compressive strength, aggregate replacement, soil stabilization, environmental treatment, bending, or vibration. Very limited attention has been given to the thermal buckling behavior of thin eco-concrete panels reinforced with cow bone particles. Therefore, the present study investigates the thermal buckling instability of thin eco-concrete panels reinforced with cow bone powder at volume fractions of 10%, 20%, and 30%. The effective thermo-mechanical properties are estimated using the FU–SHY homogenization rule. CP Theory, FSD Theory, and HSD Theory are then used to predict the critical buckling temperature. The proposed model is validated through comparison with Cheng's [21] reference results.

2 HOMOGENIZATION BY FU–SHY METHODS

The effective thermo-mechanical properties of the proposed eco-concrete were evaluated using a homogenization approach based on the FU–SHY rule. This step aims to determine the equivalent elastic, physical, and thermal properties required for the thermal buckling analysis of the panel. In this formulation, concrete is considered as a two-phase composite material consisting of an ordinary concrete matrix reinforced with cow bone powder particles. The homogenization procedure accounts for the contribution of each phase according to its volume fraction and mechanical efficiency. Such an approach is widely used for reinforced composite materials, where the effective properties depend on the matrix properties, reinforcement properties, and particle efficiency factor [27].

In the present work, the total volume of the composite is assumed to be equal to one. Therefore, the sum of the concrete matrix volume fraction and the cow bone powder volume fraction is written as follows:

$$V_C + V_{CBP} = 1 \tag{1}$$

Where V_C is the volume fraction of ordinary concrete and V_{CBP} is the volume fraction of cow bone powder particles. The effectiveness of cow bone powder reinforcement is related to the particle geometry and is determined using the ratio between the minimum and maximum particle diameters:

$$\eta = \frac{D_{min}}{D_{max}} \tag{2}$$

Where D_{min} and D_{max} are the minimum and maximum diameters of cow bone particles, respectively. The effective Young's modulus, Poisson's ratio, density, thermal expansion coefficient, and thermal conductivity are then calculated using the FU–SHY homogenization rule [27]:

$$P_H = P_C V_C + \eta P_{CBP} V_{CBP} \tag{3}$$

Where P_H is the homogenized property, P_C is the property of ordinary concrete, and P_{CBP} is the corresponding property of cow bone powder.

$$P_H = E, \nu, \rho, \alpha, \lambda \quad (4)$$

E is Young's modulus, which describes the elastic stiffness of the material and its resistance to deformation under normal stress.

ν is Poisson's ratio, which represents the ratio between lateral strain and longitudinal strain when the material is subjected to uniaxial loading.

ρ is density, which represents the mass per unit volume of the material.

α is the coefficient of thermal expansion, which describes the tendency of the material to expand or contract when its temperature changes.

λ is thermal conductivity, which measures the ability of the material to conduct heat.

The shear modulus is obtained from the classical elasticity relation.

2.1 Physico-Mathematical Model for Critical Buckling Temperature Prediction

The thermal buckling behaviour of the plate is formulated using an energy-based approach. The total potential energy of the system is expressed as:

$$\Pi = U + V_T - W \quad (5)$$

Where:

U is the elastic strain energy. V_T is the thermal potential energy. W is the work done by external loads. The total kinetic energy consists of translational and rotational contributions:

$$\mathrm{T} = T_t + t_r \quad (6)$$

The translational kinetic energy is given by:

$$T_t = \frac{1}{2}\int_V \rho(\dot{u}^2 + \dot{v}^2 + \dot{w}^2)dV \quad (7)$$

While the rotational kinetic energy is written as:

$$T_r = \frac{1}{2}\int_V \rho(\dot{\theta}_x{}^2 + \dot{\theta}_y{}^2 + \dot{\theta}_z{}^2)dV \quad (8)$$

Where:

u, v, w are the displacement components. $\theta_x, \theta_y, \theta_z$ are the rotational components.

The elastic strain energy of the structure is defined by:

$$U = \frac{1}{2}\int_V \sigma^T \varepsilon \, dV \quad (9)$$

For a homogeneous, isotropic and linearly elastic material, Hooke's constitutive law is expressed as:

$$\sigma = C\varepsilon \quad (10)$$

Thus:

$$U = \frac{1}{2}\int_V \varepsilon^T C \, \varepsilon \, dV \quad (11)$$

The strain-displacement relations are written as:

$$\varepsilon_x = \frac{\partial u}{\partial x}\,,\ \varepsilon_y = \frac{\partial v}{\partial y} \tag{12}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\,,\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x},\ \gamma_{xz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \tag{13}$$

The thermal strain vector is introduced as:

$$\varepsilon_T = \alpha \Delta T \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \tag{14}$$

Where:

ΔT is the temperature variation.

Accordingly, the thermoelastic constitutive equation becomes:

$$\sigma = C(\varepsilon - \varepsilon_T) \tag{15}$$

The thermal potential energy associated with in-plane thermal stresses is expressed as:

$$V_T = \frac{1}{2}\int_A \left[N_x^T \left(\frac{\partial w}{\partial x}\right)^2 + N_y^T \left(\frac{\partial w}{\partial y}\right)^2 + 2N_{xy}^T \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \right] dA \tag{16}$$

According to the principle of virtual work, the equilibrium condition is obtained by:

$$\delta \Pi = 0 \tag{17}$$

Which leads to:

$$\delta U + \delta V_T - \delta W = 0 \tag{18}$$

The thermal buckling condition is reached when the structure loses stability:

$$det(K + K_T) = 0 \tag{19}$$

Where:

K is the elastic stiffness matrix. K_T is the geometric stiffness matrix induced by thermal stresses.

The critical temperature variation is therefore obtained as:

$$K\Delta T_{cr} = \frac{N_{cr}(1-\nu)}{Eh\alpha} \tag{20}$$

Where:

N_{cr} denotes the critical buckling load.

h is the plate thickness.

For a simply supported rectangular plate, the classical thermal buckling temperature can be written as [21]:

$$\Delta T_{cr} = \frac{\pi^2 D(1-\nu)}{Eh\alpha} \frac{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2}{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]} \tag{21}$$

Where:

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{22}$$

a and b are the plate dimensions. m and n are the buckling mode numbers.
The displacement vector may therefore be written in compact form as:

$$u(x,y,z) = \begin{Bmatrix} u_0(x,y) - z\frac{\partial w_0}{\partial x} + f(z)\emptyset_x(x,y) \\ v_0(x,y) - z\frac{\partial w_0}{\partial y} + f(z)\emptyset_y(x,y) \\ w_0(x,y) \end{Bmatrix} \tag{23}$$

The higher-order shear function is commonly chosen as [43]:

$$f(z) = z - \frac{4z^3}{3h^2} \tag{24}$$

The use of a Higher-Order Shear Deformation Theory (HSDT) enables a more accurate representation of transverse shear effects, especially for moderately thick and thick plates subjected to thermal loading [43].

According to the First-Order Shear Deformation Theory (FSDT), also known as Mindlin plate theory, the displacement field is expressed as:

$$u(x,y,z) = \begin{Bmatrix} u_0(x,y) + z\,\emptyset_x(x,y) \\ v_0(x,y) + z\,\emptyset_y(x,y) \\ w_0(x,y) \end{Bmatrix} \tag{25}$$

In the framework of linear plate theory, the displacement field is established under the assumption of small deflections and small rotations. Therefore, the geometrical nonlinearities are neglected [21]. This assumption considerably simplifies the mathematical formulation while remaining accurate for thin and moderately thick plates subjected to moderate thermo-mechanical loading [21].

3 RESULTS AND DISCUSSION

The comparative analysis presented in Tables 1–3 highlights the capability of the proposed FU–SHY homogenization model to accurately predict the effective thermo-mechanical properties of eco-concrete reinforced with cow bone powder. The obtained numerical results are in good agreement with the reference data reported in the literature [50–52], confirming the validity and reliability of the present theoretical approach for the characterization of sustainable cementitious composites.

Table 1. Thermal conductivity comparison between present result and [50].

Cow bone powder content (%)	Reference values (W/m·°K) [50]	FU–SHY model (W/m·°K)	Difference (%)
0	1.70	1.70	0.0
5	1.55	1.58	1.9
10	1.40	1.44	2.8
15	1.25	1.30	4.0
20	1.10	1.16	5.4
25	0.95	1.01	6.3
30	0.85	0.92	8.2

Table 2. Thermal expansion coefficient between present result and [51].

Cow bone powder content (%)	Reference values ($\alpha=10^{-6}$ /°K) [51]	FU–SHY model (α=10-6 /°K)	Difference (%)
0	10.00	10.00	0.0
5	9.70	9.75	0.5
10	9.40	9.48	0.9
15	9.10	9.20	1.1
20	8.80	8.92	1.4
25	8.50	8.63	1.5
30	8.20	8.35	1.8

Table 3. Young's modulus comparison between present results and [52].

Cow bone powder content (%)	Reference values E (GPa) [52]	FU–SHY model E (GPa)	Difference (%)
0	30.00	30.00	0.0
5	29.20	29.45	0.9
10	28.30	28.75	1.6
15	27.20	27.80	2.2
20	26.00	26.65	2.5
25	24.80	25.50	2.8
30	23.50	24.30	3.4

Regarding the thermal conductivity presented in Table 1, the results indicate a progressive reduction in the effective thermal conductivity with increasing cow bone powder content. The thermal conductivity decreases from 1.70 W/m·°K for ordinary concrete to approximately 0.92 W/m·°K for the composite containing 30% cow bone powder. This behavior can be explained by the lower intrinsic thermal conductivity of cow bone particles

compared with the cementitious matrix, as well as the increase in microstructural porosity induced by the bio-mineral reinforcement. The presence of additional pores and internal discontinuities reduces heat transfer through the composite material, thereby improving its thermal insulation performance. Such a tendency is consistent with the findings reported in previous studies dealing with lightweight and bio-based concretes [50]. Furthermore, the relative deviation between the present FU–SHY model and the literature values remain below 9%, which demonstrates that the homogenization procedure captures satisfactorily the thermal interaction between the matrix and the dispersed particles.

The results reported in Table 2 concerning the effective thermal expansion coefficient also reveal a gradual decrease with increasing reinforcement content. The coefficient of thermal expansion decreases from 10×10−6/°K for the reference concrete to approximately 8.35×10−6/°K for the composite reinforced with 30% cow bone powder. This reduction can be attributed to the mineral composition of cow bone particles, which contain calcium phosphate compounds characterized by relatively stable thermal behavior. Moreover, the interaction between the cement matrix and the bio-mineral inclusions contributes to limiting the global thermal deformation of the composite material. The excellent agreement observed between the present predictions and the reference results, with deviations lower than 2%, confirms the efficiency of the FU–SHY model in reproducing the thermo-elastic response of the eco-concrete. This behavior is particularly advantageous for structures subjected to thermal loading because lower thermal expansion coefficients reduce the risk of thermal cracking and improve dimensional stability.

Concerning the mechanical behavior summarized in Table 3, the effective Young's modulus decreases progressively as the cow bone powder content increases. The elastic modulus drops from 30 GPa for ordinary concrete to approximately 24.30 GPa for the concrete containing 30% reinforcement. This reduction is mainly associated with the lower stiffness of cow bone particles and the increase in porosity generated within the composite microstructure. At higher reinforcement ratios, the interfacial transition zone between the matrix and the particles may also become weaker, leading to reduced stress transfer efficiency and lower overall rigidity. Nevertheless, the decrease remains relatively moderate, indicating that the proposed eco-concrete still preserves acceptable mechanical properties for structural and non-structural engineering applications. The maximum deviation between the present results and the literature data remains below 3.5%, which demonstrates the robustness and predictive capability of the FU–SHY homogenization approach.

Globally, the present study confirms that the incorporation of cow bone powder significantly modifies the thermo-mechanical behavior of concrete. From a thermal point of view, the reduction in thermal conductivity and thermal expansion coefficient improves the insulation capacity and thermal stability of the material. From a mechanical perspective, although a moderate reduction in stiffness is observed, the resulting elastic properties remain within acceptable engineering ranges. Therefore, the proposed eco-concrete may represent a promising sustainable construction material combining environmental benefits, waste valorization, and improved thermal performance.

The small differences observed between the present theoretical predictions and the experimental or reference results may be related to several factors not fully considered in the homogenization model, such as particle agglomeration, non-uniform distribution of reinforcement, moisture content, micro cracking effects, and imperfections at the matrix–particle interface. Despite these simplifications, the FU–SHY model provides sufficiently accurate estimations for practical engineering applications and can be considered an efficient tool for the preliminary design and optimization of bio-based eco-concretes.

The results are presented in Fig. 1. The variation of Young's modulus shows a gradual decrease with increasing cow bone powder content. This reduction remains moderate, indicating that the proposed eco-concrete maintains acceptable mechanical stiffness, with elastic modulus values remaining above 25 GPa. It is also observed that the reduction in Young's modulus becomes more noticeable at high reinforcement contents, particularly at 30% cow bone powder incorporation. This result suggests that excessive replacement levels may affect the continuity of the cementitious matrix and reduce the efficiency of stress transfer between the matrix and the reinforcement particles. Consequently, moderate replacement ratios appear to provide a better balance between sustainability and mechanical performance.

The comparison between the developed FU–SHY homogenization rule and the empirical expressions proposed by several design codes, including BSI [31], ACI [32], RBA 60 [33], BAEL 91/99 [34], Eurocode 2 [35], and BAEL 80 [36], shows good agreement. Among these references, the Eurocode 2 and ACI formulations appear to be the closest to the FU–SHY prediction. From Fig. 1, it can be observed that the FU–SHY rule provides a conservative estimation of the effective Young's modulus, with an approximate safety margin of 15%. Therefore, this homogenization approach can be considered suitable for predicting the mechanical and thermal properties of cow bone powder-reinforced eco-concrete and for introducing these properties into the subsequent thermal buckling analysis.

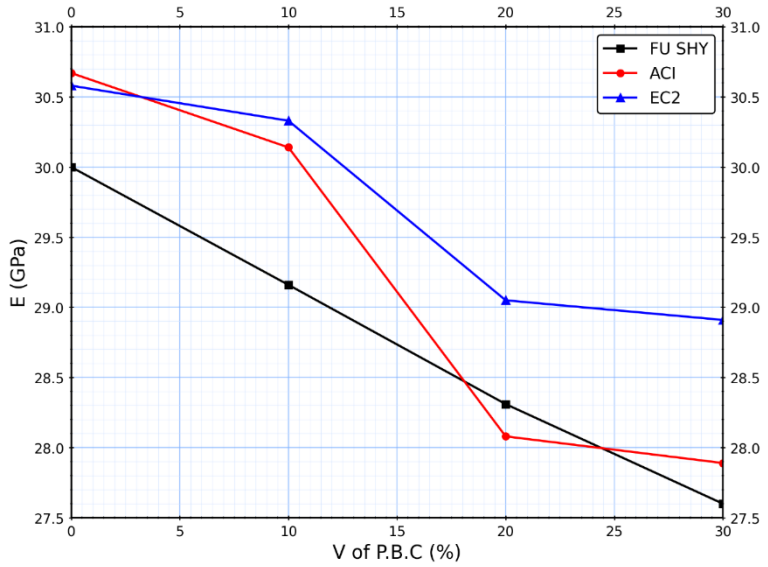

Fig. 1. Variation and comparison of the effective Young's modulus of eco-concrete reinforced with cow bone powder according to the FU–SHY homogenization rule and different international design codes.

From a thermo-mechanical perspective, the reduction in stiffness directly affects the thermal buckling resistance of the plate. Since the critical buckling temperature depends strongly on the flexural rigidity of the structure, any decrease in Young's modulus may lead to a reduction in the critical thermal load. Nevertheless, the moderate variation observed in the present study indicates that the proposed eco-concrete can still maintain acceptable thermal stability while contributing to waste valorization and sustainable construction.

Overall, the obtained results demonstrate that cow bone powder can be successfully incorporated into concrete as an eco-friendly reinforcement material. The FU–SHY homogenization rule appears to be an effective approach for estimating the equivalent

thermo-mechanical properties required for advanced structural analyses such as thermal buckling prediction.

After implementing our computational program, the obtained results were compared with those reported in the literature, particularly the study of Cheng [21] on the thermal buckling of a simply supported concrete plate. The comparison is illustrated in Fig. 2 for different geometrical configurations of the concrete plate.

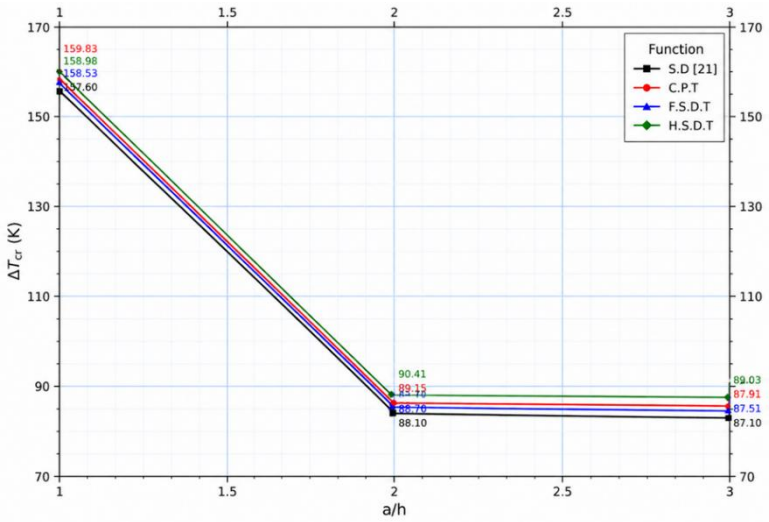

Fig. 2. Comparison of the critical thermal buckling temperature obtained using the Simple Deflection method (S.D.), Classical Plate Theory (C.P.T), First-Order Shear Deformation Theory (F.S.D.T), and Higher-Order Shear Deformation Theory (H.S.D.T) for different geometrical configurations of the concrete plate when (a=3b).

In this comparison, S.D. denotes the simple deflection method used by Cheng [21], with $\alpha = 10^{-6}\ /°K$, $\nu = 0{,}3$, and $E = 30GPa$. C.P.T refers to the Classical Plate Theory, F.S.D.T represents the First-Order Shear Deformation Theory, in which a shear correction factor is introduced to account for transverse shear effects ($K_s = 5/(6-\nu^2)$), and H.S.D.T denotes the Higher-Order Shear Deformation Theory, where the shear deformation is described by a thickness-dependent function f(z).

As shown in Fig. 2, the proposed formulations provide results that are in good agreement with the reference solution. This confirms the validity of our computational program and demonstrates the ability of the adopted plate theories to predict the critical thermal buckling temperature of concrete plates.

For the configuration a/h=30, the values predicted by C.P.T, F.S.D.T, and H.S.D.T are very close to those obtained by the Simple Deflection method. This behavior indicates that transverse shear deformation effects remain limited for relatively slender plates. Consequently, the classical theory already provides satisfactory predictions in this case.

However, for thicker plate configurations, especially when the aspect ratio changes, the influence of transverse shear deformation becomes more significant. In such cases, the F.S.D.T and H.S.D.T formulations provide more realistic predictions than C.P.T because they account for shear effects neglected in the classical plate theory. The H.S.D.T model gives slightly higher accuracy due to the higher-order distribution of transverse shear strains through the plate thickness.

It is also observed that the critical thermal buckling temperature decreases when the geometrical ratio changes from a/h=30 to a/h=40. This behavior can be explained by the reduction in structural rigidity as the plate becomes relatively thinner, making the structure more sensitive to thermal instability phenomena.

Furthermore, the influence of the geometrical parameter b/a is clearly visible. When the plate aspect ratio increases, the critical temperature decreases considerably, indicating that larger plates exhibit lower thermal buckling resistance. This tendency is physically consistent with classical stability theory for rectangular plates subjected to thermal loading.

Overall, the obtained results confirm that the developed computational model accurately predicts the thermal buckling behavior of concrete plates and that the inclusion of shear deformation effects improves the precision of the thermo-mechanical analysis, particularly for moderately thick plates. The results of the selected parameters are in good agreement with those reported by Cheng [21]. This study focuses on the effect of plate geometry and reinforcement rate through the incorporation of cow bone powder at volume fractions of 10%, 20%, and 30%.

After validating the numerical programming, the critical buckling temperature induced by thermal loading was calculated. Initially, the investigation was carried out for ordinary concrete. Then, the cow bone powder content was gradually increased. The results for ordinary concrete are presented in Fig. 3, while those corresponding to cow bone powder fractions of 10%, 20%, and 30% are shown in Figs. 4, 5, and 6, respectively.

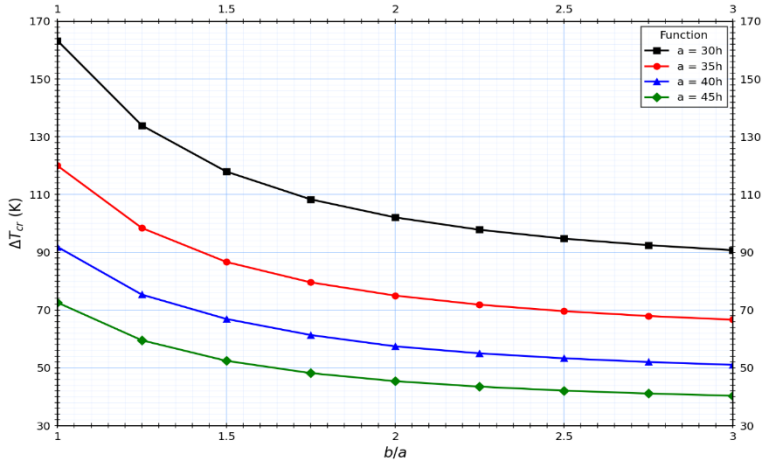

Fig. 3. Variation of the critical buckling temperature Δ_{Tcr} of ordinary concrete panels as a function of the aspect ratio b/a using H.S.D.T

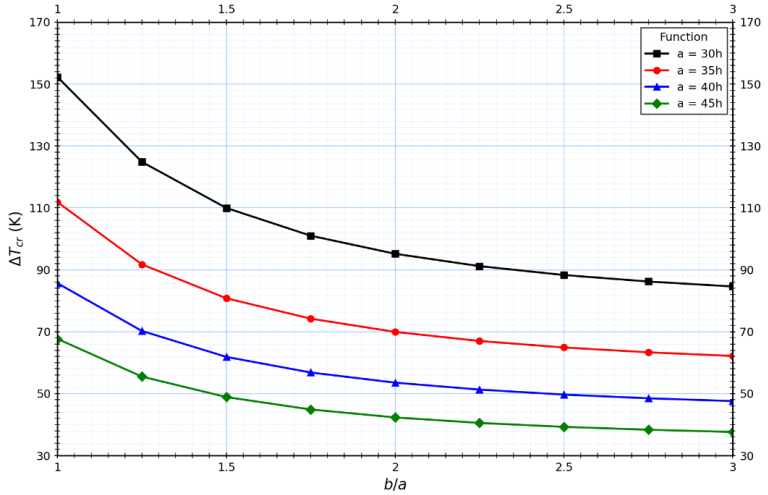

Fig. 4. Variation of the critical buckling temperature Δ_{Tcr} of eco-concrete panels reinforced with 10% cow bone powder as a function of the aspect ratio b/a using H.S.D.T.

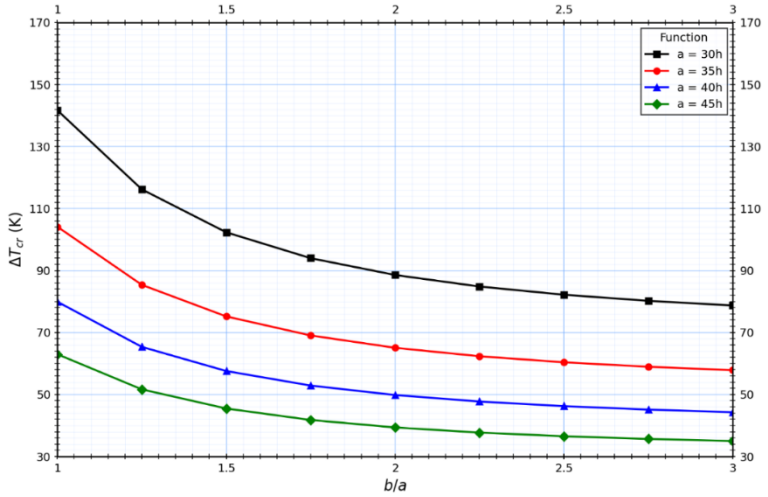

Fig. 5. Variation of the critical buckling temperature Δ_{Tcr} of eco-concrete panels reinforced with 20% cow bone powder as a function of the aspect ratio b/a using H.S.D.T.

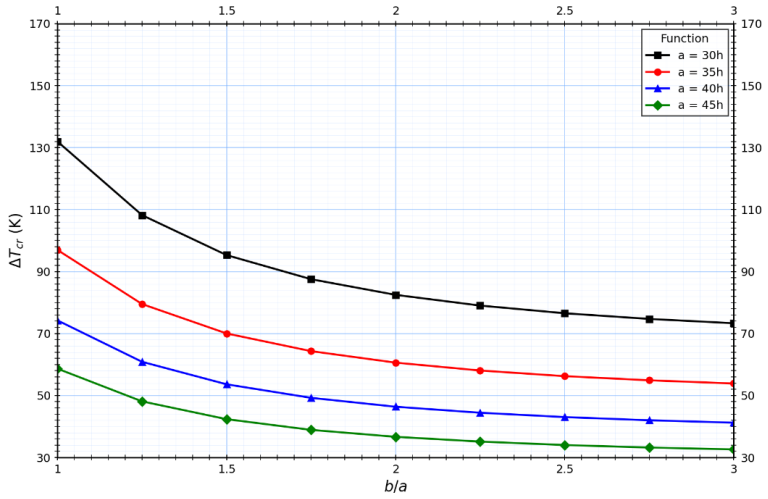

Fig. 6. Variation of the critical buckling temperature Δ_{Tcr} of eco-concrete panels reinforced with 30% cow bone powder as a function of the aspect ratio b/a using H.S.D.T.

It can be observed that increasing the two geometric ratios leads to a decrease in the critical buckling temperature, which promotes thermal instability. The first ratio is the width-to-length ratio (b/a), while the second is the length-to-thickness ratio (a/h). This behavior indicates that wider plates become more sensitive to thermal instability. Physically, increasing the plate width compared to its length reduces the structural rigidity, thereby facilitating the onset of thermal buckling under elevated temperatures. For all configurations, the curves exhibit a nonlinear decreasing tendency. A sharp reduction in ΔT_{cr} is initially observed for lower values of b/a, followed by a smoother variation for larger aspect ratios. This trend confirms the strong influence of plate geometry on thermal stability behavior. Moreover, the difference between the three plate theories remains relatively small, which may be attributed to the thin nature of the plate.

The addition of cow bone powder has an unfavorable effect on thermal stability by reducing the temperature at which buckling occurs. When calculating the ratio between the critical temperatures obtained for two successive volume fractions, a nearly constant coefficient of approximately 0.93 is obtained. The worst results are achieved for the 30% cow bone powder fraction. Since this addition is a bio-sourced material, the content is limited to 30%.

The results presented in Figs. 3–6 illustrate the variation of the critical buckling temperature ΔT_{cr} as a function of the aspect ratio (b/a) for ordinary concrete and eco-concrete reinforced with different percentages of cow bone powder.

A comparison between Figs. 3, 4, 5, and 6 reveals the significant effect of cow bone powder reinforcement. As the reinforcement percentage increases from 0% to 30%, the critical buckling temperature decreases gradually. This reduction may be attributed to the modification of the thermomechanical properties of the composite material, particularly the decrease in stiffness and thermal expansion caused by the incorporation of bio-sourced particles.

Moreover, the results indicate that the plate with 30% cow bone powder exhibits the lowest values of ΔT_{cr}, whereas ordinary concrete shows the highest thermal buckling resistance. Nevertheless, the incorporation of cow bone powder contributes to the development of sustainable and lightweight eco-concrete materials, which remain attractive for thermal insulation and environmentally friendly construction applications.

Another important observation concerns the influence of the geometric parameter a/h. Increasing the ratio a/h, corresponding to thinner plates, also leads to a reduction in the critical buckling temperature. Thin plates possess lower bending rigidity and are therefore more vulnerable to thermal instability phenomena.

4 CONCLUSION

This study investigated the thermal buckling behavior of a thin eco-concrete panel reinforced with cow bone powder particles. The main objective was to evaluate the possibility of using cow bone powder as a sustainable additive in concrete and to analyze its influence on the critical buckling temperature under thermal loading. The proposed material was treated as a homogenized composite made of an ordinary concrete matrix and cow bone powder particles. The effective mechanical, physical, and thermal properties were determined using the FU–SHY homogenization rule, and these properties were then introduced into a physico-mathematical model based on energy principles.

The results showed that the incorporation of cow bone powder modifies the thermo-mechanical behavior of concrete in a progressive and controlled manner. As the cow bone powder content increased from 0% to 30%, the effective Young's modulus, shear modulus, density, and thermal conductivity decreased, while Poisson's ratio and the thermal expansion coefficient increased slightly. These variations indicate that cow bone powder contributes to producing a lighter material with improved thermal insulation capacity, while maintaining acceptable stiffness for structural analysis. The 30% cow bone powder content provided the most significant modification of the material properties and represents a promising sustainable option, especially from an environmental point of view. The addition of cow bone powder has an unfavorable effect on thermal stability by reducing the temperature at which buckling occurs.

The thermal buckling results demonstrated that the critical buckling temperature is strongly affected by the geometry of the panel. Increasing the width-to-length ratio and the length-to-thickness ratio led to a clear reduction in the critical temperature. This means that

wider and thinner panels are more sensitive to thermal instability. The comparison between CPT, FSDT, and HSDT showed that the three theories give very close results, confirming that shear deformation has a limited effect for the thin panels considered in this work.

The validation of the developed model against Cheng's reference results showed good agreement, confirming the reliability of the proposed formulation. The model can therefore be used to predict the thermal buckling response of eco-concrete panels reinforced with cow bone powder.

The reuse of cow bone waste in concrete production offers both environmental and engineering benefits. It contributes to waste reduction, limits the exploitation of natural resources, and supports the development of eco-friendly construction materials. Future studies may extend this research by including experimental validation, fire exposure tests, durability analysis, and the study of other boundary conditions or structural configurations.

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