Creep Equations in Uniaxial Tension or Compression: An Overview

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Abstract

It is known that in practice the mechanical behavior of engineering materials in many situations is characterized by creep. In essence, this type of behavior represents an increase in strain over time under constant external loading. Time-dependent deformation of structures and facilities is also observed with increasing temperature. The creep processes have a great influence on the performance of engineering structures and facilities. These processes are of essential importance for reliability and safety. Creep has also a huge effect on structural durability. It is clear that taking into account creep is a necessary condition for obtaining realistic results when analyzing the performance of structures and facilities if they are loaded for a long period of time. The processes of creep in various materials are characterized by great diversity. This is the reason for the development of a large number of theoretical models for describing creep and for analyzing structures in the presence of creep. The key element in this analysis is the creep equation. The latter represents the relationship between stress, strain and time parameter usually under uniaxial tension or compression. The creep equations are divided into two large groups, namely linear and nonlinear. In this review paper, the basic linear creep equations are first reviewed. The corresponding linear rheological models are presented. More attention is paid to nonlinear creep equations. In these cases, the dependence between stress, strain and time is nonlinear. To describe this dependence, more general equations than those provided by linear creep models are used. Equations of stress relaxation are presented too. Both linear and nonlinear relaxation models are considered. This review paper can be used by engineers and researchers in the analysis of viscoelastic structures and facilitates.

Keywords: *Overview, Theoretical model, Creep equation, Stress relaxation, Viscoelastic structure*

1. Introduction

Ensuring the safety and durability of various facilities and structures is one of the most important goals pursued by engineers. There is a large number of factors that influence in one way or another safety and durability of facilities and structures in various sectors of engineering, including geotechnics and mining [1, 2]. Taking these factors into account in the design and calculation process is an essential prerequisite for ensuring the safe and durable operation of facilities and structures. In this regard, the theoretical models used in analyzing stresses and strains are of great importance. These models are distinguished by their diversity and varying degrees of complexity depending on the desired accuracy of the analysis and what aspects of the behavior of facilities and structures are considered in each specific case.

Very often, various facilities and structures are under load for a long period of time during their operation. It is this circumstance that gives rise to the phenomenon of creep. In principle, creep is understood as the process of gradual increase in strains over time under constant stresses [3]. Creep is one of the important aspects of the behavior of facilities and structures when they are under constant load over time. It is not difficult to assess that creep has a great effect on the safety and durability of facilities and structures [4]. Creep can cause serious damage that violates the intended design functions of facilities and structures. When the strains increase as a result of creep above a certain critical level, it can also lead to the removal of facilities and structures from exploitation. It is undeniable that analyzing creep processes and predicting their consequences are of great importance for ensuring the desired durability of facilities and structures. These analyses are performed with various theoretical models, differing in their accuracy and scope. One of the key components of these models is the creep equation. In principle, this equation defines the relationship between stresses, strains and the time factor and depends on the type of specific material. The creep equation is the basis of the theoretical models used to study stresses and strains in structures and facilities during creep. The more accurately the creep equation reflects the actual behavior of materials during creep, the more accurate the analysis results will be. Due to the significant differences in the creep processes of different materials, a large number of creep equations have been proposed in the research literature.

The purpose of this review article is to present a wide range of creep equations in uniaxial tension or compression. First, the classical linear creep models are considered. The corresponding creep equations are presented. Models under stresses that vary continuously with time are also reviewed. The corresponding dependencies between stresses, strains and the time factor are considered. A place is also given to linear rheological models for studying stress relaxation. A review of the relationships between stresses, strains and time factors for different relaxation models is made. Such relationships are also presented for models under strains varying according to different laws in time. Since in practice creep is nonlinear in most cases, nonlinear creep equations are also reviewed here. Equations for describing nonlinear stress relaxation at constant strain are also presented.

2. Overview of creep equations

First, we will review the creep equations for classical linear creep models in uniaxial tension or compression. Essentially, these are relationships between stresses, strains, and time.

We will start with the model of Maxwell. This model consists of two series-connected elements (a Hookean spring and a Newtonian dashpot). The model diagram is shown in Fig. 1. The relationship between stress, strain, and time for this model is

$$\varepsilon = \frac{\sigma}{E} + \frac{\sigma t}{\eta}. \tag{1}$$

The following notations are introduced in the above formula: ε - strain; σ - stress; E - modulus of elasticity; η - coefficient of viscosity; t - time.

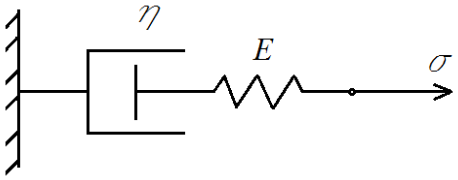

Fig. 1. Model of Maxwell.

Another classical linear creep model with two elements is that of Voigt. The essential thing about this model is that the elements are connected in parallel. The model diagram is drawn in Fig. 2.

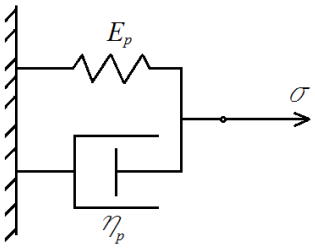

Fig. 2. Model of Voigt.

The corresponding relationship between stress, strain, and time has the form

$$\varepsilon = \frac{\sigma}{E_p}\left(1 - e^{-\frac{E_p t}{\eta_p}}\right). \tag{2}$$

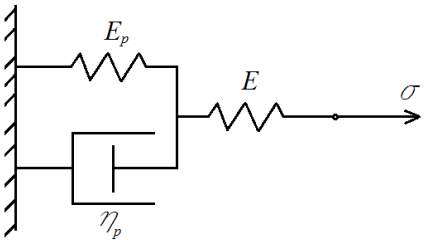

Fig. 3. Model of Kelvin.

The classical linear creep model of Kelvin is given in Fig. 3. This model has three elements. The relationship between stress, strain, and time for the model in Fig. 3 is

$$\varepsilon = \frac{\sigma}{E} + \frac{\sigma}{E_p}\left(1 - e^{-\frac{E_p t}{\eta_p}}\right). \tag{3}$$

One of the widely used classical linear creep models is the four-element model. The way the elements are connected can be seen in Fig. 4.

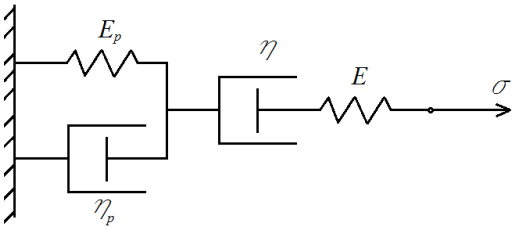

Fig. 4. Four-element creep model.

The relationship between stress, strain and time for this model is given by the formula

$$\varepsilon = \frac{\sigma}{E} + \frac{\sigma t}{\eta} + \frac{\sigma}{E_p}\left(1 - e^{-\frac{E_p t}{\eta_p}}\right). \tag{4}$$

The relationships between stresses, strains and time that have been considered so far refer to cases where the stresses applied to the creep models are constant in time. Certainly, in many situations in engineering practice the external load changes over time. This leads to the need to use creep models on which stresses are applied as functions of time.

In this regard, we will first consider the Kelvin model, when the stress applied to the model varies linearly over time, i.e.

$$\sigma = vt. \tag{5}$$

In this case, the relationship between stress, strain and time is

$$\varepsilon = \frac{vt}{E_*} + \frac{nv}{E_*}\left(1 - \frac{E}{E_*}\right)\left(1 - e^{-\frac{E_* t}{nE}}\right), \tag{6}$$

where

$$E_* = \frac{E E_P}{E + E_P}, \tag{7}$$

$$n = \frac{\eta}{E + E_P}. \tag{8}$$

Another linear creep model, loaded with a linear time-varying stress, can be seen in Fig. 5. The model has four elements. The relationship between stress, strain and time for this model is [5]

$$\varepsilon = \frac{\sigma}{\theta^2 t}\left(\frac{1}{\eta_L} - \frac{\beta}{\theta}\right)\left(e^{-\theta t} - 1\right) + \frac{\beta\sigma}{2\theta}t + \frac{\sigma}{\theta}\left(\frac{1}{\eta_L} - \frac{\beta}{\theta}\right) + \frac{\sigma}{E_H}, \tag{9}$$

where

$$\theta = \frac{E_B}{\eta_L}\left(1 + \frac{\eta_L}{\eta_D}\right), \tag{10}$$

$$\beta = \frac{E_B}{\eta_D \eta_L}. \tag{11}$$

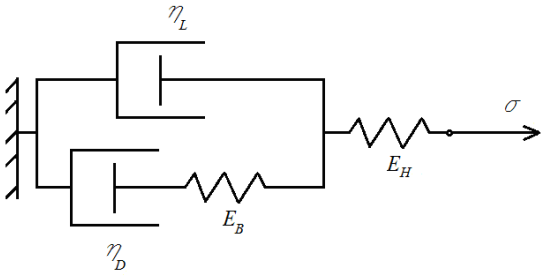

Fig. 5. Creep model loaded with a linear time-varying stress.

In engineering practice, continuous changes in loads on facilities and structures over time are possible according to a variety of laws. For example, here we consider the case of a stress varying over time according to an exponential law

$$\sigma = \sigma_0 e^{ft}. \tag{12}$$

This stress is applied to the four-element model in Fig. 4. Then the relationship between stresses, strains and time is [6]

$$\varepsilon = \frac{\sigma_0}{E} e^{ft} + \frac{\sigma_0}{\eta f}\left(e^{ft} - 1\right) + \lambda\left(e^{ft} - e^{-\alpha_1 t}\right), \tag{13}$$

where

$$\alpha_1 = \frac{E_P}{\eta_P}, \tag{14}$$

$$\alpha_2 = \frac{\sigma_0}{\eta_P}, \tag{15}$$

$$\lambda = \frac{\alpha_2}{f + \alpha_1}. \tag{16}$$

There are a number of cases in which the loads vary according to periodic laws in time. This requires the use of creep models that are under the action of periodic stresses. Figure 6 shows a scheme for a possible periodic change in stresses. The dependence between stress, strain and time for the Voigt creep model when the stress changes according to the scheme in Fig. 6 has the form [7]

$$\varepsilon = \frac{\sigma}{E_P} - \frac{1}{E_P}\left[\sum_{j=1}^{\infty} B_j \cos\left(j\overline{\omega}t\right) + \sum_{j=1}^{\infty} D_j \sin\left(j\overline{\omega}t\right) \right] + \sum_{j=1}^{\infty} P_j \cos\left(j\overline{\omega}t\right) + \sum_{j=1}^{\infty} R_j \sin\left(j\overline{\omega}t\right), \tag{17}$$

where

$$B_j = \frac{2\sigma_\alpha}{Tj\overline{\omega}}\left\{\sin\left(j\overline{\omega}fT\right) + \beta\left[\sin\left(j\overline{\omega}T\right) - \sin\left(j\overline{\omega}fT\right)\right]\right\}, \tag{18}$$

$$D_j = \frac{2\sigma_\alpha}{Tj\overline{\omega}}\left\{1 - \cos\left(j\overline{\omega}fT\right) + \beta\left[\cos\left(j\overline{\omega}fT\right) - \cos\left(j\overline{\omega}T\right)\right]\right\}, \tag{19}$$

$$P_j = \frac{\delta\lambda_j - \mu_j j\overline{\omega}}{j^2\overline{\omega}^2 + \delta^2}, \tag{20}$$

$$R_j = \frac{\mu_j}{\delta} + \frac{\left(\delta\lambda_j - \mu_j j\overline{\omega}\right)j\overline{\omega}}{\delta\left(j^2\overline{\omega}^2 + \delta^2\right)}, \tag{21}$$

$$\overline{\omega} = \frac{2\pi}{T}, \tag{22}$$

$$\lambda_j = \frac{B_j}{\eta}, \tag{23}$$

$$\mu_j = \frac{D_j}{\eta_P}, \tag{24}$$

$$\delta = \frac{E_P}{\eta_P}, \tag{25}$$

$$\beta = \frac{\sigma_\beta}{\sigma_\alpha}, \tag{26}$$

$$f = \frac{T_\alpha}{T}. \tag{27}$$

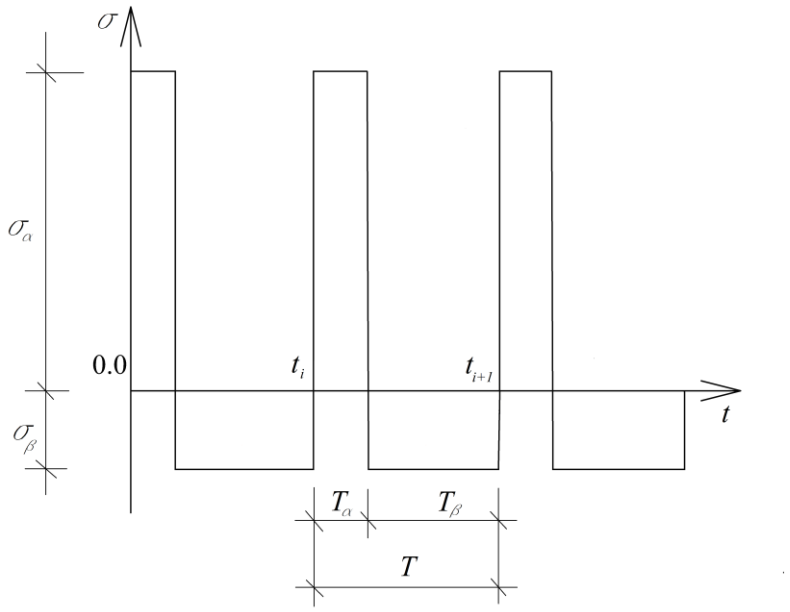

Fig. 6. Scheme for a periodic change in stresses.

Classical models can, certainly, also be used to analyze stress relaxation under constant strain. For example, if a constant strain, ε , is applied to the Maxwell model, the relationship between stress, strain, and time is expressed by the formula

$$\sigma = E\varepsilon e^{-\frac{Et}{\eta}}. \tag{28}$$

The stress relaxation in the Kelvin model under constant strain is given by the expression

$$\sigma = E_*\varepsilon + \left(E - E_*\right)\varepsilon e^{-\frac{(E+E_P)t}{\eta_P}}. \tag{29}$$

The study of stresses under different laws of strain variation over time is also of interest. For example, the relationship between stresses, strains and time for the model in Fig. 7, when the strain varies linearly, i.e. $\varepsilon = vt$, has the following form

$$\sigma = \frac{\mu\varepsilon}{\lambda t}\left(1 - e^{-\lambda t}\right), \tag{30}$$

where

$$\lambda = \frac{E}{\eta_Q}, \tag{31}$$

$$\mu = E\left(1 + \frac{\eta_R}{\eta_Q}\right). \tag{32}$$

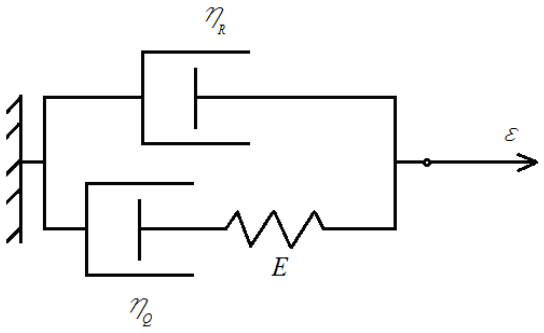

Fig. 7. Rheological model under linearly varying strain.

When a strain varying according to the law

$$\varepsilon = \theta t^2 \tag{33}$$

is applied to the Kelvin model, the relationship between stresses, strains and time is given by the formula

$$\sigma = E\left(\varepsilon - \varepsilon_2\right), \tag{34}$$

where

$$\varepsilon_2 = \psi_5\left(1 - e^{-\psi_1 t}\right) + \psi_3 t^2 + \psi_4 t\,, \tag{35}$$

$$\psi_1 = \frac{E + E_P}{\eta_P}, \tag{36}$$

$$\psi_2 = \frac{E\theta}{\eta_P}, \tag{37}$$

$$\psi_3 = \frac{\psi_2}{\psi_1}, \tag{38}$$

$$\psi_4 = -2\frac{\psi_2}{\psi_1^2}, \tag{39}$$

$$\psi_5 = -2\frac{\psi_2}{\psi_1^3}. \tag{40}$$

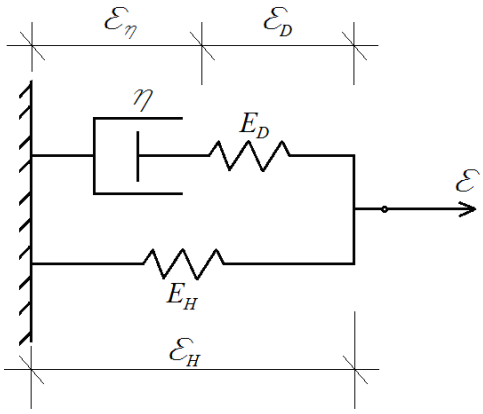

Fig. 8. Three-element model.

The model in Fig. 8 is under a strain varying according to a sine law

$$\varepsilon = \varepsilon_0 \sin\left(\omega t\right). \tag{41}$$

In such a case, the relationship between stresses, strains and time is [8]

$$\sigma(t) = \eta\left[\ \theta\beta e^{-\beta t} + \lambda\omega\cos\left(\omega t\right) - \theta\omega\sin\left(\omega t\right)\ \right] + E_H\varepsilon_0\sin\left(\omega t\right), \tag{42}$$

where

$$\beta = \frac{E_D}{\eta}, \tag{43}$$

$$\delta = \beta\varepsilon_0, \tag{44}$$

$$\lambda = \frac{\delta\beta}{\omega^2 + \beta^2}, \tag{45}$$

$$\theta = -\frac{\delta\omega}{\omega^2 + \beta^2}. \tag{46}$$

Although the creep models reviewed so far differ from each other in the number and manner of connecting their elements, as well as in the type of loading, there is one circumstance that is the same for these models. It is that the relationships between stresses, strains, and time are linear. However, in practice, creep processes in various facilities and structures are very often nonlinear, which means that the relationships between stresses, strains, and time are nonlinear functions. Therefore, here are presented the main nonlinear creep equations (i.e., nonlinear relationships between stresses, strains, and time), often applied in various nonlinear creep analyses.

First, we will consider the following modification of the Ramberg-Osgod law, used in nonlinear creep mechanics [9]:

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H_C}\right)^{\frac{1}{n_C}}, \tag{47}$$

where H_C depends on time according to the formula

$$H_C = \frac{1}{\left(D_3\, t^{\phi}\right)^{\frac{1}{\delta}}}. \tag{48}$$

Besides,

$$n_C = \frac{1}{\delta}. \tag{49}$$

Here D_3, ϕ and δ are parameters.

Another modification of the Ramberg-Osgod law (46) is that in which the time dependence of the H_C is given by the five-parameter function [10]

$$H_C = \left[B_1 + B_2 t + B_3\left(1 - e^{-\beta t}\right)\right]^{-\frac{1}{m}}. \tag{50}$$

Nonlinear relationships between stresses, strains and time, which are also applicable to describe nonlinear creep, include the following two-parameter logarithmic or hyperbolic functions of time [11]

$$\varepsilon = \frac{\sigma}{E} + D_4 \log\left(1 + \beta_4 t\right), \tag{51}$$

$$\varepsilon = \frac{\sigma}{E} + \frac{D_5 t}{1 + \beta_5 t}. \tag{52}$$

Dependencies involving a three-parameter power function of time are also used in nonlinear creep mechanics [11].

$$\varepsilon = \frac{\sigma}{E} + D_3 \sigma^{\delta} t^{\phi} \tag{53}$$

Nonlinear relationships between stresses, strains and time are also used, in which two power functions of the stress are included [11].

$$\varepsilon = \frac{\sigma}{E} + B\sigma^m t + D\sigma^{\alpha}. \tag{54}$$

In nonlinear creep mechanics, nonlinear relationships between stresses, strains, and time are also used, which include both power and exponential functions of the type [9]

$$\varepsilon = \frac{\sigma}{E} + B\sigma^m t + D\sigma^{\alpha}\left(1 - e^{-\beta t}\right). \tag{55}$$

Nonlinear creep equations of the following type are also used [12]:

$$\varepsilon = \frac{\sigma}{E} + \frac{t}{\eta}\sigma\, e^{\sigma^{\frac{3+\gamma}{3m}}}. \tag{56}$$

The nonlinear creep equations presented so far in this review paper are characterized by the fact that the relationship between

stress and strain at the initial time, i.e. at $t = 0$, is linear (this is the first term on the right-hand side of equations (46), (50) - (55)). However, there are creep equations in which the relation between stress and strain at the initial time is also nonlinear. Such is the following five-parameter creep equation [13]:

$$\varepsilon = A_0 sh(\beta_0 \sigma) + A^n t^n sh(\beta \sigma). \tag{57}$$

At the end of this review, a stress-strain-time relationship is presented, describing the nonlinear stress relaxation under constant strain. This non-linear relationship involves three parameters and has the following form [9]:

$$\sigma = \frac{E\varepsilon}{\left[tBE(m-1)\sigma^{m-1} + 1\right]^{\frac{1}{m-1}}}. \tag{58}$$

3. Conclusion

A review of the relationships between stresses, strains and time in uniaxial tension or compression for engineering materials, the mechanical behavior of which under continuous loading is characterized by creep of strains, is presented. First, the classical linear creep models are considered. They are characterized by the fact that they are built from two types of elements, namely Hookean springs and Newtonian dashpots. By using a different number of elements connected in series or parallel, a wide variety of linear models can be obtained that describe the characteristic features of creep in different materials and engineering structures. In order to assess the suitability of these linear models and their corresponding creep equations for use in each specific case, a comparison should be made with data from experimental studies of the creep process. Linear creep models under stresses that change continuously in time are also reviewed. In this case, a variety of combinations of elements are also possible, which allows models to be built whose dependences between stresses, strains and time are as close as possible to those obtained from experimental studies of creep. Classical rheological models are also presented to describe the relaxation of stresses under constant strains over time. Here, a variety of combinations of elements can also be made, which allows for obtaining relaxation equations that correspond as closely as possible to data from experimental studies of the relaxation process for each specific case. In principle, linear models and the resulting dependences between stresses, strains and time have limited applicability, since in most cases creep processes are nonlinear. In this regard, a number of nonlinear dependences between stresses, strains and time have been obtained. The most commonly used nonlinear dependences are reviewed here. Modifications of the Ramberg-Osgood law applied to the analysis of nonlinear creep are presented. Nonlinear relationships between stresses, strains and time in the form of logarithmic, hyperbolic, power and exponential functions are also considered. Nonlinear creep equations are also given, including a relatively large number of parameters. In all cases, however, the application of these nonlinear relationships necessarily undergoes verification by comparison with data from creep tests. In conclusion, I will note that a promising direction for sharply increasing the accuracy of the analysis of creep processes in facilities and structures is the digitalization of the nonlinear relationships between stresses, strains and time obtained on the basis of creep tests. The digitized relationships can be applied in combination with efficient numerical methods for simulating the behavior of engineering facilities during nonlinear creep.

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