Study of Mechanical Stress in an Area Near the Cutting Edge of Standard Teeth Used to Determine the Displacement Forces During the Machining of Coal and Rock

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Abstract

In this paper, we attempt to establish the stress distribution in an area near the cutting edge of a standard tooth using elasticity theory and numerical methods, when the stress forces are known from experiments. To improve the design and quality of the cutting teeth on the bucket wheels of excavators used for coal extraction in open pits, it is necessary to know the forces acting on them during the excavation process. These forces can be determined under laboratory conditions using standard teeth, or directly on the excavator using actual teeth (when the machine is operating under real-world conditions). In both cases, the mechanical stresses in the cutting teeth are determined and then converted into forces. Although the second option is more accurate, due to its complexity and the high financial cost required to procure the necessary equipment, the first option was chosen.

Keywords: cutting teeth, shear strength of rocks, displacement forces, elasticity theory method, finite element analysis, Von Misses stress chart

1. Introduction

To improve the design and quality of the cutting teeth on the bucket wheels of excavators used for coal extraction in open pits, it is necessary to know the forces acting on them during the excavation process. For this, knowledge of the maximum stress occurring near the cutting edge of the standard teeth is necessary, to determine (by analogy) the geometric characteristics, materials, treatments (thermal, chemical, and mechanical) required for the actual teeth that equips bucket-wheels of excavators use for coal extractions in open-pits.

In order to determine the shear strength of rock and coal by first measuring the cutting forces under laboratory conditions (on samples taken from various mining fields), a tensometric bar was designed and constructed, which was successively equipped during the cutting process with cutting teeth having different geometric characteristics (rake angle, clearance angle, point angle, side angle, etc.). With each tooth, a predetermined number of cut grooves of varying depths and in different directions relative to the stratification were made on the coal and rock samples, simultaneously recording the tangential, normal, and lateral components F_{x_0}, F_{y_0} and F_{z_0} of the resultant force on the tooth, relative to three rectangular directions with respect to the cut surface. The lateral components of the resultant force (F_{z_0}), due to the constraints imposed by laboratory conditions, were neglected in the following study, so that the problem was reduced to determining a plane stress state in the reference tooth, based on which the external forces on the cutting tooth's surface (the external loads) corresponding to the given displacements can be determined, using the equations of elasticity theory.

The stress state in a detached element near the cutting edge of the tooth under study was also determined using the finite element method (F.E.M.), when the tooth is loaded with the the experimentally determined tangential and normal forces (F_{x_0} and F_{y_0}) and a comparation was made between the results obtained using the equations of elasticity theory and those obtained with F.E.M.

The theoretical and methodological background of the present study is based on previous research in the fields of mining machinery, rock cutting mechanics, elasticity theory, finite element analysis, and numerical modelling methods [1–10].

2. The determination of the plane stress state in an area near the cutting edge of the standart tooth using the equations of elasticity theory and by finite element method (F.E.M.)

2.1 Theoretical aspects; The determination of the plane stress state using the equations of elasticity theory

Fig. 1.a and 1.b show the assembly consisting of the strain gauge bar 1 and, respectiely, the standard tooth 2 (which has the smallest sharpening angle of 32^0) from the set of 5 standard teeth used during the experiments, its design, and the forces acting on it during cutting.

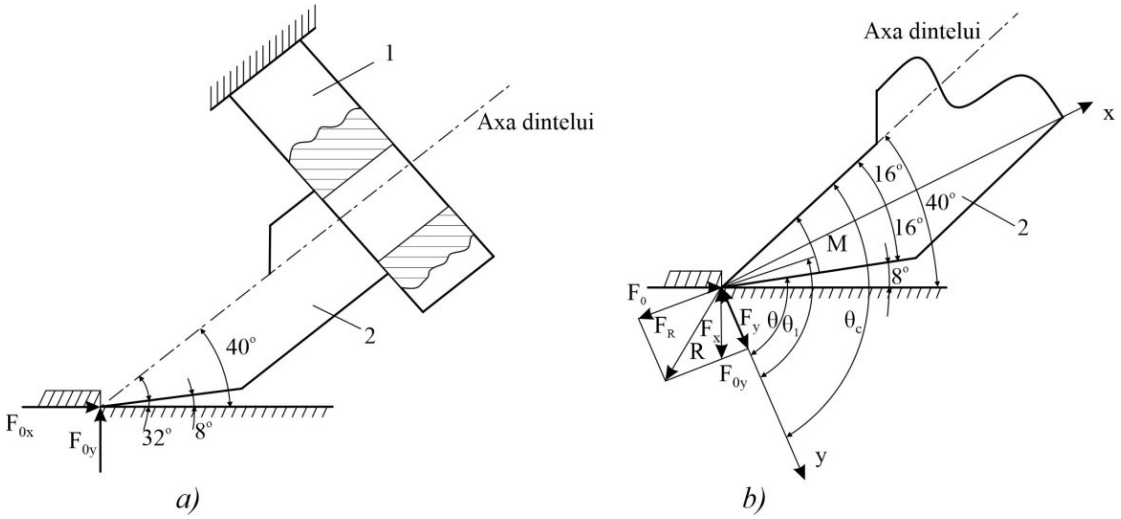

Fig. 1. The strain-gauging bar and the standard tooth assembly (1.a) and the geometrical parameters of the standard tooth with the components of the load-forces during the cutting process (1.b)

In Fig. 1, the following are denoted: F_{x_0} and F_{y_0} – the experimentally determined tangential and normal forces; F_x and F_y – the resultants of the forces F_{x_0} and F_{y_0} in the x and y directions (the x-axis representing the bisector of the rake angle).

The helix angle of 40^0 and the rake angle of 8^0 of the tooth were maintained for all reference teeth.The solution to the Flamand–Boussinescq problem, established for the case of a concentrated force $\vec{R}(\vec{F_x}, \vec{F_y})$ acting on an elastic medium bounded by a plane, which extends indefinitely below this plane, can also be applied to the tooth tip, treated as a wedge.

Fig. 2 shows an OBC element detached from near the tooth cutting edge, loaded with the force $\vec{R}(\vec{F_x}, \vec{F_y})$, where:

-2α - the rake angle of the tooth; θ_1, θ_2 *and* θ - the angles between the wedge surfaces and the y-axis, and between the line defining a point $M(x, y)$ on the wedge and the y-axis, respectively.

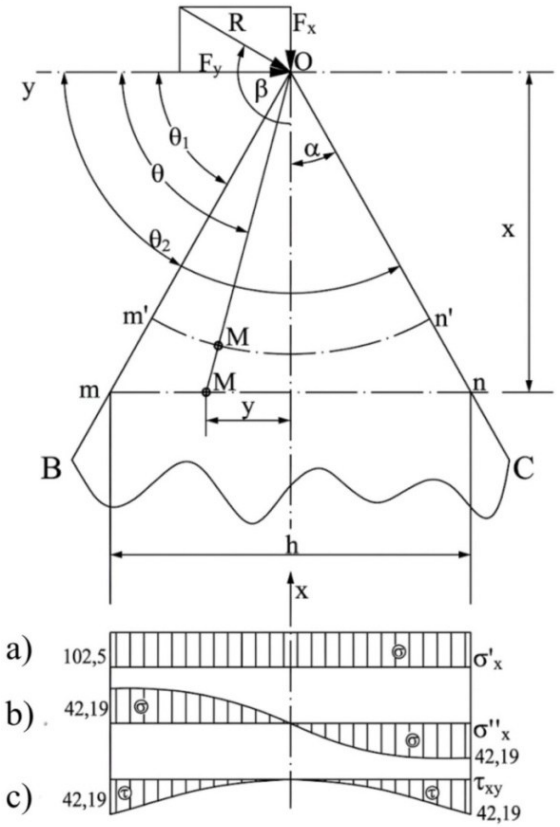

Fig. 2. Detached element near the tooth cutting edge

Across the width of the wedge of size b, measured perpendicular to the plane of Fig. 2, the components F_x and F_y represent the resultants of uniformly distributed loads p_x and p_y , respectively:

$$F_x = p_x \cdot b; F_y = p_y \cdot b \tag{1}$$

From the loading with force F_x, the stress in section mn, in the case of a narrow wedge (with a small angle α), is:

$$\sigma_x' = \frac{2F_x}{b(2\alpha + sin2\alpha)\cdot x} \tag{2}$$

From the loading of the beam with force F_y, the stresses at point M in section mn are:

$$\left.\begin{aligned} \sigma_x^{"} &= -\frac{F_y\cdot x\cdot y}{I_z}\left(\frac{tg\alpha}{\alpha}\right)^3\cdot sin^4\theta, \\ \tau_{xy} &= -\frac{12F_y\cdot y^2}{b\cdot h^3}\left(\frac{tg\alpha}{\alpha}\right)^3\cdot sin^4\theta \end{aligned}\right\}, \tag{3}$$

where h is the height of the mn section, $I_z = \frac{b\cdot(2x\cdot tg\alpha)^3}{12}$ is the moment of inertia of the mn section about the z-axis, and $h = 2x\cdot tg\alpha$.

From relations (3) it can be seen that these stresses are zero along the wedge axis (for $y = 0$) and have maximum values at the edge of the mn section. However, if instead of the flat mn section the wedge is sectioned by a cylindrical section $m'n'$ with center at O, then in this section we will have only normal stresses.

Thus, at a point M' on the arc $m'n'$ we have only radial normal stresses:

$$\left.\begin{aligned} \sigma_r &= -\frac{2R}{\pi}\cdot\frac{cos(\theta-\beta)}{r} \\ \tau_{r\theta} &= \sigma_\theta = 0 \end{aligned}\right\}, \tag{4}$$

where the angle ($\theta - \beta$) is the polar angle (Fig. 2), measured starting from the direction of the load R, and $r = CM'$ is the polar distance to the point M'.

After substitution, relations (3) become:

$$\left.\begin{aligned} \sigma_x^{"} &= -\frac{F_y\cdot x\cdot y}{8b\cdot x^3\cdot tg^3\alpha}\left(\frac{tg\alpha}{\alpha}\right)^3\cdot sin^4\theta = -\frac{3F_y\cdot y}{2b\cdot x^3\cdot\alpha^3}\cdot sin^4\theta, \\ \tau_{xy} &= -\frac{12F_y\cdot y^2}{b\cdot x^3 tg^3\alpha}\left(\frac{tg\alpha}{\alpha}\right)^3\cdot sin^4\theta = -\frac{12F_y\cdot y^2}{b\cdot x^3\alpha^3}\cdot sin^4\theta \end{aligned}\right\}, \tag{5}$$

Results obtained with elasticity theory equations:

The forces F_x *and* F_y are obtained from Fig.1(for the reference tooth with a sharpening angle of 32^0):

$$\left.\begin{aligned} F_x &= F_{x_0}\cdot cos\,24^0 + F_{y_0}\cdot sin24^0 \\ F_y &= -F_{x_0}\cdot sin\,24^0 + F_{y_0}\cdot cos24^0 \end{aligned}\right\} \tag{6}$$

Using expressions (6) and taking into account that $y = x\cdot tg(90^0 - \theta)$, relations (2) and (5) become:

$$\left.\begin{aligned} \sigma_x' &= -\frac{2(F_{x_0}\cdot cos\,24^0 + F_{y_0}\cdot sin24^0)}{b\cdot(2\alpha+sin2\alpha)\cdot x} \\ \sigma_x^{"} &= -\frac{3\cdot(-F_{x_0}\cdot sin\,24^0 + F_{y_0}\cdot cos24^0)\cdot tg(90^0-\theta)}{2b\cdot x\cdot\alpha^3}\cdot sin^4\theta \\ \tau_{xy} &= -\frac{3\cdot(-F_{x_0}\cdot sin\,24^0 + F_{y_0}\cdot cos24^0)\cdot tg(90^0-\theta)}{2b\cdot x\cdot\alpha^3}\cdot sin^4\theta \end{aligned}\right\} \tag{7}$$

We note that the values of the forces F_{x_0} and F_{y_0} acting on the tooth-strain gauge tensometric bar assembly were determined during the tests using the resistive electrical strain gauge method; these values were then substituted into equation (7) to further determine the stresses in the cutting tooth that occur during the machining process.

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Regarding the above, for the values $F_{x_0} = 4000N$ and $F_{y_0} = 2000N$, obtained when cutting a groove with a depth of $h = 20mm$, from relations (6) we obtain: $F_x = 4453N$ and $F_y = 387N$.

For a tooth width $b = 40mm$, equations (7) become:

$$\left.\begin{aligned} \sigma_x' &= -\frac{205}{x} \\ \sigma_x^{"} &= -344{,}65\frac{tg(90^0-\theta)}{x}\cdot sin^4\theta \\ \tau_{xy} &= -344{,}65\frac{tg(90^0-\theta)}{x}\cdot sin^4\theta \end{aligned}\right\} \tag{8}$$

where, for the tooth in question,θ takes values from $\theta_1\ to\ \theta_2$, i.e., $\theta = 74^0$ ... 106^0.

Figures 2, a, b, and c respectively show the variations of the stress $\sigma_x', \sigma_x^{"}\ and\ \tau_{xy}$ in the section mn located at a distance $x = 2mm$ from the tooth tip, resulting from relations (8).

The expression for the equivalent stress according to IV-2 strength theory (Von Mises) at point M as a function of θ is:

$$\sigma_e = \frac{1}{x}\sqrt{[205 - 344{,}65tg(90^0-\theta)\cdot sin^4\theta]^2 + [344{,}65tg(90^0-\theta)\cdot sin^4\theta]^2} \tag{8}$$

Fig. 3 shows the variation of the equivalent stress in the section mn, located at a distance $x = 2mm$ from the tooth tip, as a function of the angle θ.

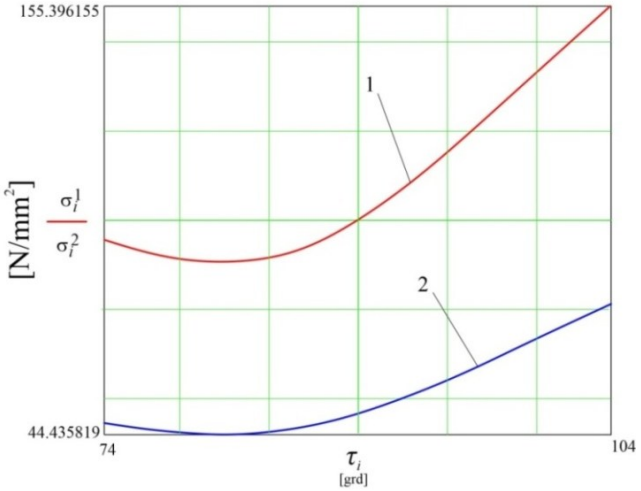

Fig. 3. Graphical representation of the equivalent stress σ_e as a function of by the angle θ for $x = 2\ mm$ (curve 1) and $x = 4\ mm$ (curve 2)

Since the standard teeth are made of improved OLC45 steel, with $\sigma_{02} = 380 \ldots 420\ N/mm^2$, it follows that for the forces obtained during the measurements, we have $\sigma_e < \sigma_{02}$, and a maximum safety factor $c = 2.3$ for $x = 2mm$.

Substituting into equation (8) $\sigma_e = \sigma_{02}$, it follows that the lower limit of the yield stress is achieved for $x \leq 0{,}42mm$.

2.2. Stress determination using the Finite Element Method (F.E.M)

The stress state in a detached element near the cutting edge of the tooth under study was also determined using the finite element method, when the tooth is loaded with the forces F_{x_0} and F_{y_0}.

Fig.4 shows the discretization of the detached tooth element into 3,502 SHELL 3 elements, and Fig. 5 presents the stress state according to von Mises theory for this element.

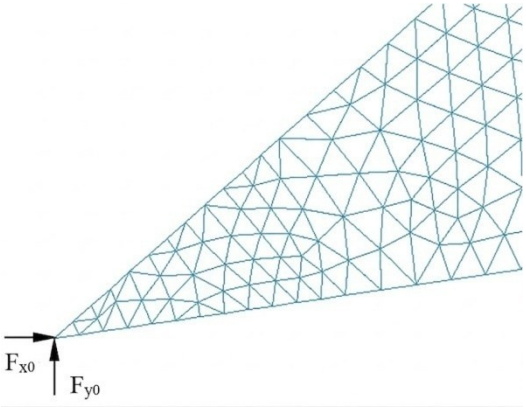

Fig. 4. The mesh of the detached tooth element

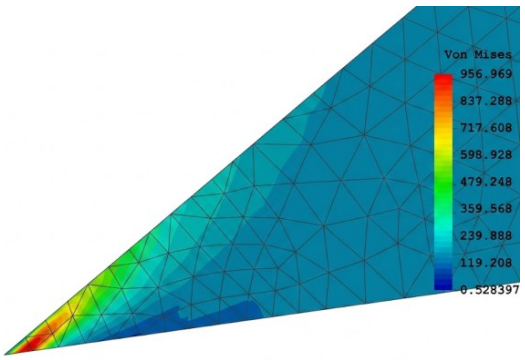

Fig. 5. Stress chart [Von Mises]

The graph in Fig. 6 shows the variation of stress along the bisector of the cutting edge, over the interval 2…4 mm, relative to the tooth tip.

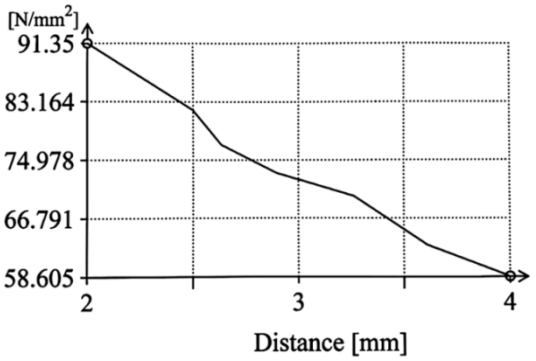

Fig. 6. Stress variation along the bisector of the tooth cutting edge in the range of 2...4 mm

Fig. 3 and Fig. 6 show that at a distance of $2mm$ from the tip of the standard tooth, measured along the bisector, the stress values determined by the methods of elasticity theory and the finite element method are $91.45\ N/mm^2$and $91.35\ N/mm^2$, respectively, which confirms the accuracy of the calculation.

3. Conclusions

The study of stress values near the cutting edge of a reference tooth represents an application of known relationships from the theory of elasticity to a concrete case with a relativily high occurance in the engineering practice.

The approach to this topic arose from the need to conduct cutting tests on rocks and coal under laboratory conditions.The study, developed based on the methods of elasticity theory, was supplemented by the finite element method, with the differences between the results obtained by the two methods being insignificant.

The methodology used to address the topic of this paper can also be applied to other types of cutting teeth used on planers, jointer, and mortisers.

References

1. Kunze, G. et al. Baumaschinen – Erdbau- und Tagebaumaschinen. Springer Vieweg, Berlin, 2012
2. Sant'Anna, A.; Pereira, T. Predictive maintenance using hardness-based structural integrity indicators in mining components. International Journal of Mining Science and Technology, 2019
3. McKyes, E., Soil Cutting and Tillage, Elsevier, Amsterdam, 1985
4. Sadowski, T.; Marsavina, L. Dynamic hardness testing methods and applications in engineering materials. Journal of Materials Testing, 2016
5. Pearson, C.E. – Theoretical Elasticity, Harvard University Press, Cambridge, Mass 1959
6. Bathe, K. J., Finite Element Procedures, Prentice Hall, New Jersey, SUA, 1996
7. Dassault Systèmes, Abaqus 2025 Documentation – Scripting User's Guide, Dassault Systèmes Simulia Corp., Providence, Rhode Island, SUA, 2025
8. Sultanov L.U., Numerical Modelling of Deformation of Soil Foundations. Procedia Earth and Planetary Science 15 (2015) pg 119-124
9. Sultanov L.U., Davydov, R.L., 2014 Mathematical modeling of large elastic-plastic deformations Applied Mathematical Sciences, 60(8): 2991-2996
10. Davydov, R.L., Sultanov, L.U., 2013b Numerical algorithm of solving the problem of large elastic-plastic deformation by FEM, PNRPU Mechanics Bulletin, 1 pg 81-93