System for Generating Pure Torsion in Delaminated Multilayered Bars under Impact Action of External Loading

Victor Rizov

Department of Technical Mechanics, University of Architecture, Civil Engineering and Geodesy, 1 Chr. Smirnensky blvd., 1046 – Sofia, Bulgaria, E-mail: v_rizov_fhe@uacg.bg; ORCID: 0000-0002-0259-3984

https://doi.org/10.29227/IM-2026-02-02-047

Submission date: *2026-05-04* | Review date: *2026-21-04*

Abstract

The basic aim of this paper is to present a system for generating pure torsion in multilayered bars with delamination under impact action of external loading. This system can be used in experimental investigations of delamination in multilayered bars subjected to pure torsion under impact loading. In fact, the system transforms the impact load caused by a falling striker into a torsion moment applied to the front end of the bar. The system represents a lever mechanism consisting of a rigid horizontal lever, a rigid vertical rod and two circular discs. The rod is connected by a joint to the front end of the lower arm of the delamination in the multilayered bar. The lever is connected by a joint also to front end of the upper arm of the delamination. The two circular discs are connected by joints to the lever. In addition, the discs are also connected to each other with a thread. The impact is caused by a striker that falls downward, sliding along the right vertical branch of the thread. The striker is a cylindrical body with a hole at its centroid. The thread passes through this hole so that the striker can slide down the thread. A theoretical analysis of the bar with delamination is performed. In this regard, a solution of the strain energy release rate (SERR) in the multilayered bar under pure torsion caused by the impact is presented. The SERR is found as a function of the torsion moment applied on the front end of the bar. To verify the solution of SERR, an approach known from specialized literature is used. The behavior of SERR when changing the torsion moment and some geometrical and material parameters of the lever system and the multilayer bar is studied. This paper can serve as a theoretical basis for developing an experimental setup to study delamination in multilayered bars under pure torsion caused by impact loading.

Keywords: *Torsion caused by impact loading, System for converting impact load to torsion moment, Delamination, Multilayered bar, Theoretical analysis*

1. Introduction

In our time, the development of engineering, including construction, is closely linked to the widespread use of rational constructions made of artificially created materials with a layered structure [1, 2, 3, 4]. The most commonly used are multilayered beams, bars, plates and shells. Layered structures of the "sandwich" type are also attracting the attention of engineers and constructors. The interest in multilayer structural materials currently shown by civil engineers is provoked by the very good properties of these materials, such as high strength and relatively low weight. Multilayer structures also have high resistance to atmospheric and chemical influences. Essentially, multilayer materials are combinations of different layers that are bonded together by adhesion. The layers in a multilayer material can be arranged in such a way as to achieve the fullest possible use of their strength. However, to ensure the safe functioning of structures made of multilayer materials, it is of great importance to guarantee excellent adhesion between the layers. The breakdown of adhesion between layers is known as delamination.

In essence, delamination is a type of damage to the integrity of the multilayered materials, expressed in the separation of layers. It is known from engineering practice that delamination creates great risks for the normal operation of structures and equipment. Delamination is one of the factors leading to the shortening of the life of multilayer structures. In addition, delamination is also a prerequisite for catastrophic failure of structures, especially under the impact action of external loads. Therefore, in multilayer structural materials, one of the main issues is the study of delamination due to various forces and influences [5, 6, 7, 8].

The main objective of this paper is to present a system for converting an impact load from a falling striker into a torsional moment loading a multilayered bar with a delamination crack in pure torsion. This system includes a lever mechanism and two circular discs. The latter are connected to each other by a thread having two vertical branches. The upper end of the left vertical branch of the thread is fixedly attached. A striker slides down the right vertical branch of the thread and strikes a washer firmly attached to the lower end of the thread, thus creating an impact load. This load is converted to a torsion moment applied at the front end of the multilayered bar. SERR is derived in the bar loaded by impact torsional moment. An approach from the literature is applied to check the SERR. The behavior of the SERR when changing some basic geometric and material parameters of the lever system and the bar is analyzed.

2. Analysis of the System

A sketch of the studied bar is depicted in Fig. 1. There is a delamination crack in this bar. As already noted, our main goal here is to present a system for loading the bar in pure torsion under the impact action of an external load. The system actually represents a lever mechanism attached to the front end of the bar (Fig. 1). This mechanism has a rigid horizontal lever, CDH, and a rigid vertical rod, BC. The lever is connected to the rod by a joint in point, C. The lower end of the vertical rod is connected to the lower arm of the delamination by a joint in point, B. In addition, the horizontal lever is connected by a joint at point, D, to the

upper arm of the delamination (Fig. 1). A moment of magnitude, M, is applied to the lever as can be seen in Fig. 1. It is obvious that this moment loads the two arms of the delamination and the intact part of the bar in pure torsion.

Practically, the moment on the lever is generated using two circular disks as shown in Fig. 2. These disks are connected to the lever using joints at points, L_1 and L_2. A thread is passed through both discs as can be seen in Fig. 2. The upper end, Q, of the left vertical branch of the thread is fixedly fixed. A rigid washer is firmly attached to the lower end, R, of the right vertical branch of the thread.

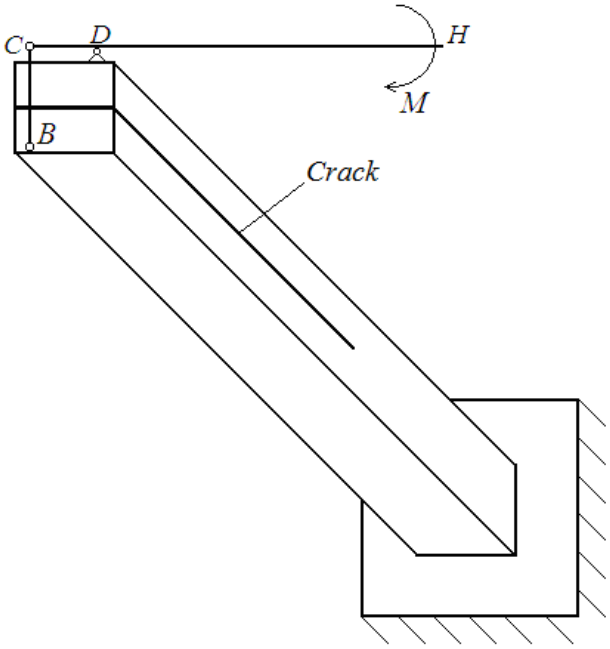

Fig. 1. Sketch of the delaminated multilayered bar and the lever mechanism.

The impact load is generated using a striker, which is a cylindrical body with its own weight, F. There is a hole in the cylindrical body along the centroid. The right vertical branch of the thread passes through the hole, so that the striker slides down the thread and hits the washer. An impact load is generated by dropping the striker down the thread from a height, h_F. This load induces a moment in the horizontal lever. The magnitude of the moment assuming static application of F can be derived by Eq. (1).

$$M = Fl, \tag{1}$$

where l is the distance between the two vertical branches of the thread (Fig. 2).

The theoretical analysis of the bar with delamination in Fig. 1 is related to determining the SERR, G, in pure torsion under impact loading. In this regard, we apply Eq. (2).

$$G = \frac{\partial U}{b\partial a}, \tag{2}$$

where b is the bar cross-section width, a is the delamination length, U is the strain energy,. The latter is derived by Eq. (3).

$$U = U_{lw} + U_{up} + U_{in}, \tag{3}$$

where

$$U_{lw} = a\left(\sum_{j=1}^{j=n_{lw}} \iint_{(A_j)} u_{0lwj} dA\right), \tag{4}$$

$$U_{up} = a\left(\sum_{j=1}^{j=n_{up}} \iint_{(A_j)} u_{0upj} dA\right) \tag{5}$$

and

$$U_{in} = \left(l_{br} - a\right)\left(\sum_{j=1}^{j=n} \iint_{(A_j)} u_{0inj} dA\right) \tag{6}$$

are the strain energies in the lower and upper arm of the delamination, and the intact part of the bar, u_{0lwj}, u_{0upj} and u_{0inj} are the corresponding specific strain energies, l_{br} is the bar length. As specified in the introduction to the current paper, the studied bar is multilayered (the parameters of its cross-section are defined in Fig. 3). Therefore, the strain energies are found by summing over layers in Eqs. (4), (5) and (6).

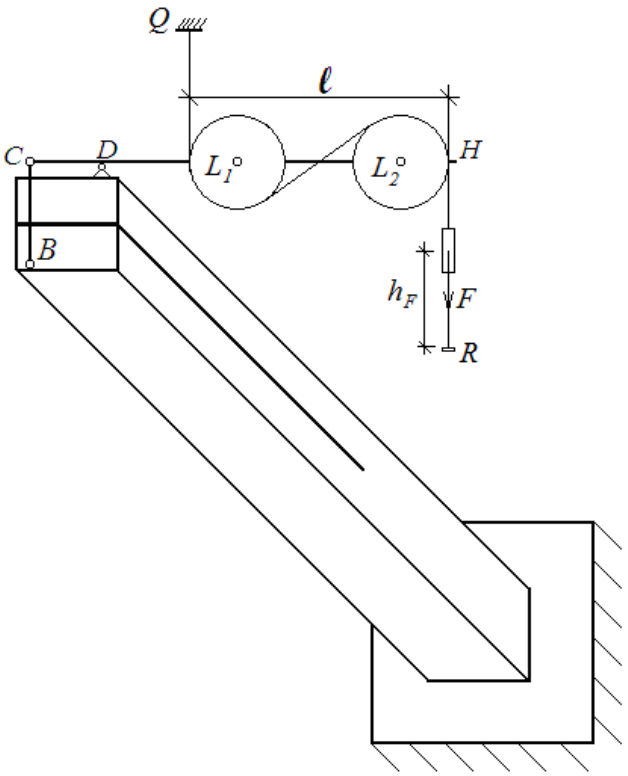

Fig. 2. Scheme for converting impact load into torque using two circular disks.

Equation (7) is applied to find u_{0lwj}.

$$u_{0lwj} = \frac{\tau_{lwdj}^2}{2G_{*j}}, \tag{7}$$

where G_{*j} is the angular deformation modulus of the considered layer of the bar. Tangential stress, τ_{lwdj}, from the torsion of the bar under the impact action of the external load is found using the dynamic coefficient, k_{dyn}, according to Eq. (8).

$$\tau_{lwdj} = \tau_{lwsj} k_{dyn}, \tag{8}$$

where τ_{lwsj} is the tangential stresses from static torsion of the bar with moment, M.

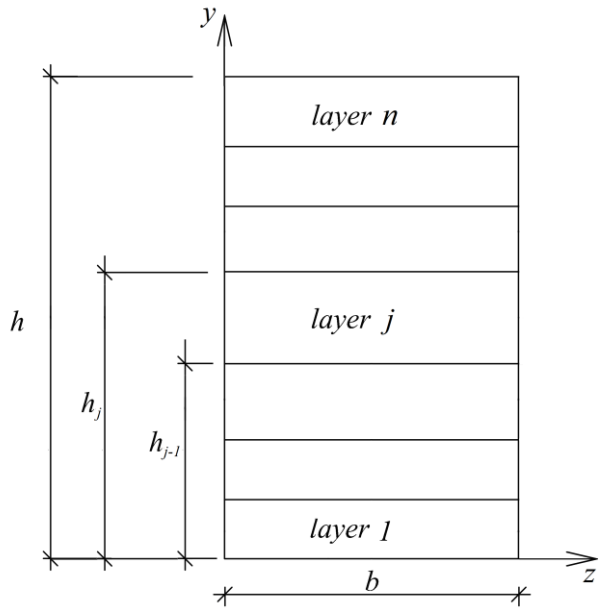

Fig. 3. Sketch of the multilayered bar cross-section.

The procedure for determining τ_{lwsj} is described by Eqs. (9) – (19) [5].

$$\tau_{lwsj} = \frac{M_{lw}}{S_1}\left\{ \left[\sum_{k=1,3,\ldots}^{\infty} \left(P_{k,j} ch\alpha_k y + Q_{k,j} sh\alpha_k y\right)\alpha_k sh\alpha_k z \right]^2 + \right.$$

$$\left. + \left[\sum_{k=1,3,\ldots}^{\infty} \left(P_{k,j} sh\alpha_k y + Q_{k,j} ch\alpha_k y + \frac{8G_{*j}b^2}{k^3\pi^3} \right)\alpha_k \cos\alpha_k z \right]^2 \right\}^{\frac{1}{2}}, \tag{9}$$

where

$$M_{lw} = \frac{M}{S_1} S_{lw}, \tag{10}$$

$$\alpha_k = \frac{k\pi}{b}, \tag{11}$$

$$P_{k,j} = \frac{2}{\left(g_{j,j+1}-1\right) sh2\alpha_k h_j}\left[Q_{k,j+1} g_{j,j+1} + Q_{K,j}\left(sh^2\alpha_k h_j - g_{j,j+1} ch^2\alpha_k h_j \right) + \right.$$

$$\left. + g_{j,j+1}\left(r_{k,j+1} - r_{k,j}\right) ch\,\alpha_k h_j \right), \tag{12}$$

$$P_{k,j+1} = \frac{2}{\left(g_{j,j+1}-1\right) sh2\alpha_k h_j}\left[Q_{K,j+1}\left(ch^2\alpha_k h_j - g_{j,j+1} sh^2\alpha_k h_j \right) - \right.$$

$$\left. - Q_{k,j} + \left(r_{k,j+1} - r_{k,j}\right) ch\,\alpha_k h_j \right), \tag{13}$$

$$g_{j,j+1} = \frac{G_{*j}}{G_{*(j+1)}}, \tag{14}$$

$$r_{k,j} = \frac{8G_{*j}b^2}{k^3\pi^3}. \tag{15}$$

$$P_{k,1} = -\frac{Q_{k,1} ch\alpha_k h_0 + r_{k,1}}{sh\alpha_k h_0}, \tag{16}$$

$$P_{k,n} = -\frac{Q_{k,n} ch\alpha_k h_n + r_{k,n}}{sh\alpha_k h_n}. \tag{17}$$

$$S_1 = \frac{8}{\pi^2} \sum_{k=1,3,\ldots}^{\infty} \frac{1}{k^2} \left\{ \frac{\alpha_k}{2} \sum_{j=1}^{n} r_{k,j} \left(h_j - h_{j-1}\right) + \right.$$
$$+ \sum_{j=1}^{n} P_{k,j} sh \frac{\alpha_k \left(h_j + h_{j-1}\right)}{2} sh \frac{\alpha_k \left(h_j - h_{j-1}\right)}{2} +$$
$$\left. + \sum_{j=1}^{n} Q_{k,j} sh \frac{\alpha_k \left(h_j - h_{j-1}\right)}{2} ch \frac{\alpha_k \left(h_j + h_{j-1}\right)}{2} \right\}. \tag{18}$$

$$S_{lw} = \frac{8}{\pi^2} \sum_{k=1,3,\ldots}^{\infty} \frac{1}{k^2} \left\{ \frac{\alpha_k}{2} \sum_{j=1}^{n_{lw}} r_{k,j} \left(h_j - h_{j-1}\right) + \right.$$
$$+ \sum_{j=1}^{n_{lw}} P_{k,j} sh \frac{\alpha_k \left(h_j + h_{j-1}\right)}{2} sh \frac{\alpha_k \left(h_j - h_{j-1}\right)}{2} +$$
$$\left. + \sum_{j=1}^{n_{lw}} Q_{k,j} sh \frac{\alpha_k \left(h_j - h_{j-1}\right)}{2} ch \frac{\alpha_k \left(h_j + h_{j-1}\right)}{2} \right\}. \tag{19}$$

The axes, y and z, and the distance, h_j, are defined in Fig. 3.

The procedure for obtaining u_{0upj} and u_{0inj} is analogous to that for u_{0lwj}. Therefore, we use Eqs. (7) – (19) by making the necessary substitutions.

To determine k_{dyn} we apply the approach presented in [9]. This leads to Eq. (20).

$$k_{dyn} = 1 + \sqrt{1 + \frac{2h_F}{\varphi_{st}\left(\frac{b}{2} + DH\right)}}, \tag{20}$$

where φ_{st} is the twist angle of the front end of the bar from static torsion with moment, M.

To find φ_{st} we apply Eq. (21).

$$\varphi_{st} = \frac{\partial U_{st}}{\partial M}, \tag{21}$$

where U_{st} is the strain energy in bar loaded in static torsion with moment, M.

The U obtained by Eq. (3) is substituted into Eq. (2) to find the SERR (in this regard, the Matlab is used).

To verify the SERR thus found, an approach known from specialized literature is used [10]. This approach is based on Eq. (22).

$$G = \frac{1}{2b} M_{dyn}^2 \frac{\partial C_M}{\partial a}, \tag{22}$$

where

$$M_{dyn} = Mk_{dyn}, \tag{23}$$

$$C_M = \frac{\varphi_{dyn}}{M_{dyn}}, \tag{24}$$

$$\varphi_{dyn} = \varphi_{st} k_{dyn}. \tag{25}$$

Calculations using Eq. (22) confirm what was obtained using Eq. (2).

3. Parametric Study

To gain a clearer picture of the behavior of a multilayered bar with a delamination loaded in pure torsion under impact loading conditions, the results of a parametric study of SERR, obtained based on the theoretical analysis from Section 2 of the paper, are presented here in dimensionless graphical form. The parametric study is performed for $h = 0.11$ m, $b = 0.12$ m, $l_{br} = 1.00$ m, $a = 0.60$ m, $n = 4$, $n_{lw} = 2$, $n_{up} = 2$ and $F = 50$ N.

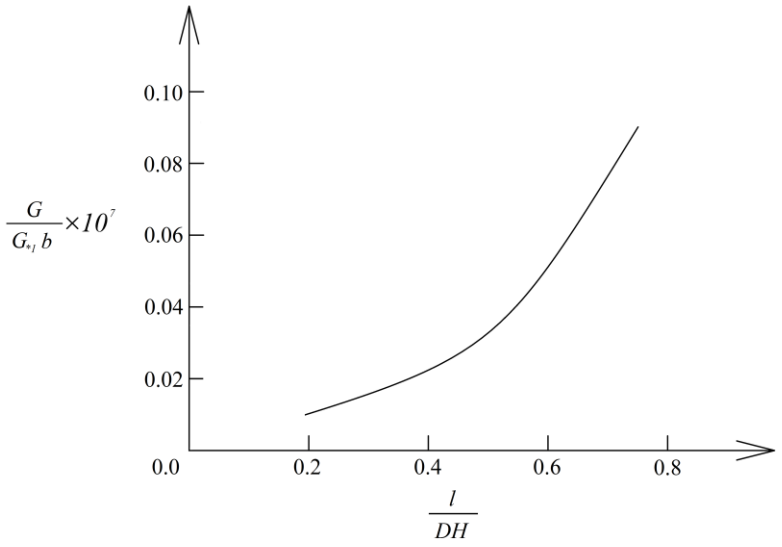

Fig. 4. Relation between the l/DH ratio and the SERR.

One of the studied parameters of the lever system is the ratio l/DH (this ratio has a direct relationship with the impact torsion moment acting on the multilayered bar). Therefore, the study of the relation between the above-mentioned ratio and the SERR in the bar is of great importance for clarifying the operation of the lever system. The change of the dimensionless SERR with increasing ratio, l/DH, is depicted in Fig. 4. It can be seen in Fig. 4 that by changing the ratio, l/DH, we are able to vary the impact torque applied to the delaminated bar.

The thickness-to-width ratio, h/b, characterizing the cross-section of the multilayered bar, is the next parameter under study. In this regard, the change in the SEER with an increase in the ratio h/b is analyzed. The graph obtained in this analysis is presented in Fig. 5. It can be observed that the change in h/b ratio has a significant influence on SERR under impact torsion (Fig. 5).

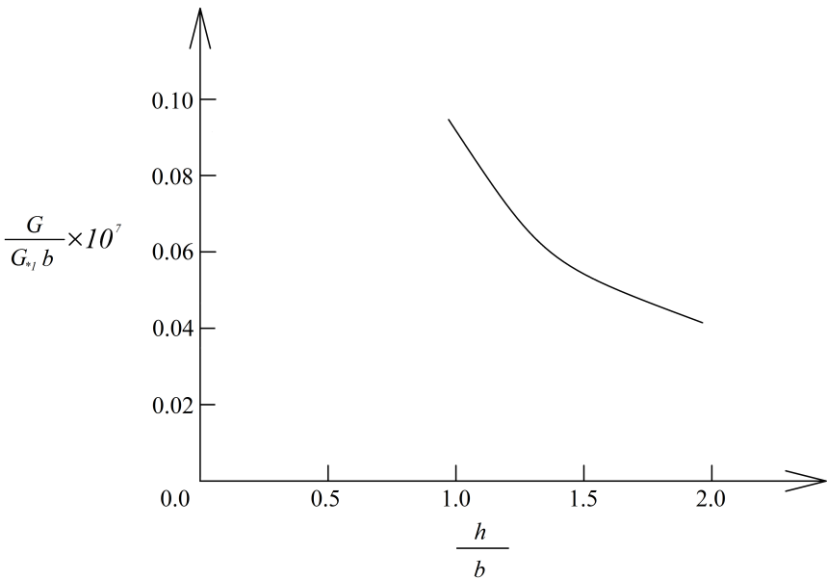

Fig. 5. Relation between the h/b ratio and the SERR.

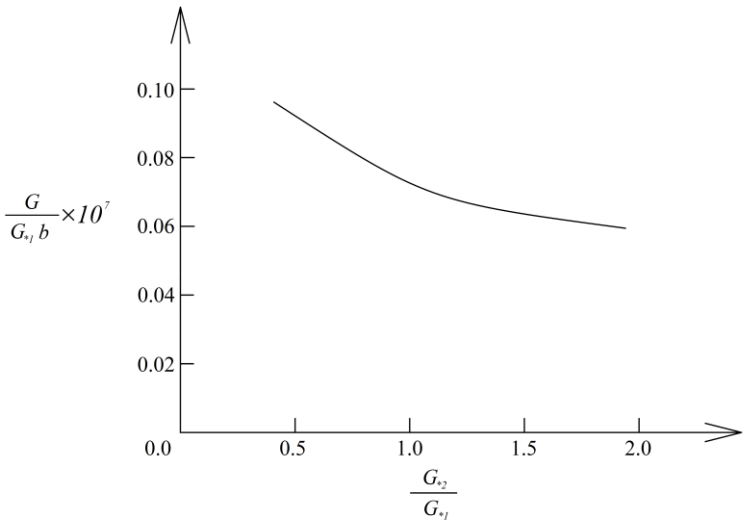

Fig. 6. Relation between the G_{*2}/G_{*1} ratio and the SERR.

Similarly, a study is also conducted on the dependence of SERR on the G_{*2}/G_{*1} ratio. The change in SERR with increasing G_{*2}/G_{*1} ratio is shown in the graph in Fig. 5. The reason for the decrease in the SERR as the G_{*2}/G_{*1} ratio increases is that the torsional stiffness of the multilayered bar increases.

4. Conclusion

A system for converting the impact load, realized by a falling striker, into a pure torsion moment, applied to a delaminated multilayer bar, is theoretically analyzed. The main idea implemented in the system is to use a cylindrical striker with a hole in its centroid. A vertical thread runs through the hole so that the striker can slide down the thread and strike a rigid washer attached at the lower end of the thread. The impact load thus realized is transformed into a moment by the thread, which passes through two cyircular disks connected by joints to a horizontal lever. The latter transmits this moment through a vertical rod and a joint to the front end of the two arms of the delamination crack, loading the multilayered bar in pure impact torsion. The objective of the analysis is to obtain a solution of SERR in the delaminated multilayered bar loaded with pure impact torsion using the system for converting the impact load described above. An approach from the specialized literature is applied to check the obtained solution. The parametric study carried-out by using the solution gives an idea of the relationship between the SERR and some parameters (such as ratios l/DH, h/b and G_{*2}/G_{*1}) of the lever system and the delaminated bar loaded in impact torsion. The performed analysis gives reason to assume that the lever mechanism for converting impact load into torsional moment can be used in systems for experimental testing of delamination in multilayer bars under impact torsion.

Acknowledgments

The current research project under contract BN351/26 is financially supported by the Center for Scientific Research and Design (CSRID) at UACEG.

References

1. S. J. Lloyd and J. M. Molina-Aldareguia, “Multilayered materials: a palette for the materials artist”, Phil. Trans. R. Soc. Lond. A, 361, 2931-294 (2003).
2. Y. Ivanov and V. Stoyanov, “High Technologies and New Construction Materials in Civil Engineering,” in Education, Science, Innovations. (Proc. Ist Int. Conf. of the European Polytechnical University, June 9-10, 2011), pp. 161-169.
3. X. Wang, Y. Sun and K. Liu, “Chemical and structural stability of 2D layered materials”, 2D Materials, 6, 042001 (2019).
4. V. I. Rizov, “Delamination analysis of multilayered beams exhibiting creep under torsion”, Couple. Syst. Mech. 10, 317-331 (2021).
5. I. Minchev, Mechanics of layered materials (Technique, 1996).
6. N. A. Dolgov, “Analytical Methods to Determine the Stress State in the Substrate – Coating System Under Mechanical Loads”, Strength of Materials 48, 658-667 (2016).
7. N. Dowling, S. Kample and M. Kral, Mechanical Behavior of Materials: Engineering Methods for Deformation, Fracture, and Fatigue (5th Edition, Pearson, 2019).
8. V. I. Rizov, “Inhomogeneous beam structures of rectangular cross-section loaded in torsion: A delamination study with considering creep”, Procedia Struct. Integr. 41, 94-102 (2022).
9. B. V. Zaslavsky, Strength of materials (Science, 1986).
10. V. I. Rizov, Mechanics of composite materials (BIAP, 2005).