The Multifactorial Approach to Power Quality Analysis in Underground Mining

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Abstract

This research presents the development of a multifactorial static multiplicative model for analysing power quality in underground mining power systems. The objective is to synthesize a generalized indicator of power quality by integrating key parameters such as voltage dips and sags, frequency deviations, harmonic distortion, and other critical indicators that influence the energy efficiency and reliability of the electrical network. The proposed model structure was developed using the synthesis method, with its parameters identified through a maladaptive approach based on the least squares method. To validate the model's accuracy, mathematical statistics techniques were employed. As a result, mathematical relationships were derived to evaluate a generalized power quality index using data on voltage drop, frequency deviation, and harmonic distortion. The model, characterized as static and multiplicative, requires full-spectrum quality data for parameter identification via a non-adaptive approach. Comparative accuracy analysis between a single-factor model and the proposed three-factor model revealed a correlation coefficient of 0.951 for the former and 0.923 for the latter. While the multifactor model demonstrates a 2.94% reduction in statistical accuracy, both models qualify as having "very high" reliability according to the Chaddock scale. This confirms the practical applicability of the multifactor approach in real-world mining energy systems. The scientific novelty lies in the improved multifactor model structure that synthesizes multiple quality indicators into a unified framework. Its practical value is evident in applications for managing power flow within industrial microgrids in underground mines, particularly those integrating local power generation sources.

Keywords: *power quality, underground mining, multifactor model, voltage dips, energy efficiency, static multiplicative model, maladaptive identification, mine power systems, least squares method, power flow management*

INTRODUCTION

The industrial mining complex for the extraction of iron ore (IO), which serves as the primary raw material for metallurgical production, is a system-forming sector that significantly contributes to Ukraine's GDP and foreign exchange reserves [1]. This industry plays a vital role in supporting the national economy and ensuring industrial stability [2]. Thanks to the output of enterprises within this sector, Ukraine consistently ranks among the top ten leading mining countries in the world [3]. The high-quality iron ore extracted by these enterprises is in demand both domestically and internationally. As a result, the iron ore industry remains a strategic asset in strengthening Ukraine's economic position on the global stage [3, 4].

However, assessing the prospects for the further functioning of these types of enterprises, provided that the process of increasing the cost of iron ore extraction is stable, this positive can be levelled to the level of drama [5]. One of the odious factors of the negative impact on this complex and basic format of the economy of extraction of the analysed type of minerals is energy intensity, or rather the electric energy intensity of extraction, which in its constant creative growth dominates the cost of IO extraction [3].

It should be noted that the current measures implemented by mining enterprises to address this issue have not yielded the desired results – the electric energy intensity of iron ore (IO) mining continues to grow [6]. This trend reflects not only the natural impact of existing mining technologies and the progressive increase in mining depths but also several other critical factors. These include inefficiencies within the energy management systems and structural shortcomings in the power supply infrastructure of the enterprises [3,7]. From a management perspective, this indicates a need for a more integrated and strategic approach to energy use [7, 8]. Enhancing energy efficiency must become a priority within enterprise development plans to ensure long-term sustainability and cost-effectiveness.

When characterizing the possibilities of implementing measures to introduce energy-efficient measures into the practice of mining enterprises, it is appropriate to state that the potential of canonical in format and perfect in time of application of measures has already been largely exhausted [3, 9]. The next priority in the ranking of currently priority and most effective measures is the process of creating modern power supply systems (PSS) with the inclusion in the structure of their functioning of the process of managing power flows within the intra-industrial energy complex: power supply – power consumption as a micro-variant of the microgrid concept [3, 4, 10].

However, both in the first and second variants of the components of the solution to the problem, there are unsolved problems, the dominant among which is the problem of power quality [11]. And if in the first case this issue has been considered historically for a long time, then in the second – only in the starting variant. At the same time, the impact of creating synergistic microgrids on power quality has been statistically proven [12].

Analysis of known and available scientific research in the direction of increasing the energy efficiency of mining industries in general and mining industries, with their specifics, it should be noted that there is a small level of correlation between the need for this direction for the economy of the relevant industry and the state of the search.

In this work [13], the general state of energy efficiency of mining enterprises was analysed, and options for "road maps" for the implementation of the proposed scientific projects were presented.

In studies [14, 15], based on the specifics of the specifics of the functioning of mining enterprises with an underground mining method of IO and the corresponding options for using both natural and own energy resources, a certain range of variable solutions for their implementation in the practice of the analysed types of production is outlined. These works analyse the effectiveness of creating new structures based on existing PSS of mining enterprises – with distributed power generation. However, the problem of EE quality in these types of mining production was not sufficiently analysed in the specified searches. However, it is the complex of EE quality components that is the basis that has the unspent potential for increasing the energy efficiency of these enterprises, since the negative im-pact of the actual state of these indicators on the final option, increasing energy efficiency, is currently the most influential on the state of this indicator and, so far, the least implemented.

An interesting positive example for domestic scientists in the analysed direction is the experience of foreign colleagues [16–19]. However, evaluating the range of research by foreign scientists and outlining the technology of domestic production, we note that there is a set of specifics here, which is characteristic of the technology of functioning and the corresponding regime of operation of the complex: power supply – power consumption of domestic underground mining enterprises [20].

Therefore, targeted search is necessary in achieving the goal. One of the levers of insufficient solution to the problem of searching for improving the quality of EE is the lack of an effective scientific base for conducting scientific search in sufficient volume.

The main method of modelling the power quality in electric power systems is the model-oriented approach, which is inherent in the MATLAB software complex. Examples of works based on this option are [21] and [22]. Its use is characterized by the simplicity of constructing and analysing electrical and electromechanical complexes, particularly the impact of their operation on power quality indicators. This is explained by the presence of repeatedly tested block libraries in the system, such as Simulink, Simscape Power Systems, and analysis tools, such as powergui. However, this option for analyzing quality indica-tors has disadvantages. The user is not aware of the internal organization of the blocks, as well as the simulation process itself. An even greater problem, in our opinion, is the need to build your own system for modeling any quality indicator, which does not allow for a comprehensive study and does not allow tracking the degree of correlation of many factors on power indicators. So, in work [21], a whole set of models for such analysis is created, which includes models of a short circuit in a distribution line and starting an asynchronous motor, which are used to simulate voltage dips; capacitor bank switching model, lightning surges to simulate pulsed transients; non-linear consumer models to simulate non-sinusoidality; electric arc furnace model used to simulate voltage disturbances. The same can be seen in [22].

Considering the above, several scientific studies have been conducted that investigate the comprehensive assessment of the impact of a number of factors on the power quality. Analytical methods are widely used in this regard.

Thus, in [23], a model of power quality assessment was investigated, based on the method of hierarchy analysis, which, based on individual factors, such as voltage deviation, harmonics, and asymmetry, determines the generalized power quality in the power grid. Moreover, a certain factor has its own weight. The proposed approach reduces the influence of subjectivity.

The unmodified hierarchical process analysis method was also used by the authors in [24] to assess power quality in systems with a large share of renewable energy sources. The article introduces a unified power quality index for assessing power quality indicators, both in individual sections and in the entire power grid.

The work [25] is devoted to the determination of the power quality using the method (TOPSIS), which combines the meth-od of hierarchy analysis (AHP), inter-criteria correlation (CRIT-IC) and the entropy weight method (EWM). The first deter-mines the subjective weight of factors, and the others – the objective weight. The method allows to assess the impact of various factors on the power quality. The inter-criteria correlation method (CRITIC) is also used in [26] to assess the power quality.

Search [27] complements the considered method of hierarchy analysis by adding fuzzy logic. The authors consider such a quality indicator as voltage sag. It is determined that it is influenced by the characteristics of the power grid and consumers. Moreover, the weight of each of the characteristics is determined by subjective and objective methods. The method of hierarchical process analysis gives a subjective assessment of the importance of the factor, and the method of maximizing deviations – an objective one. By weighing the complex weight of the indicators, the probability of voltage sag on the power grid section is established.

The use of fuzzy logic, but already together with cloud theory for multifactorial assessment of power quality is proposed in [28]. This method is aimed at reducing the influence of subjectivity.

Statistical and probabilistic methods are also used in assessing power quality.

Thus, in [29] a model was created for a comprehensive assessment of power quality in a system with distributed generation using multifactorial variance analysis. For each section of the power network, the authors determine the weight of each quality indicator and the factors that affect it. This will make it possible to focus on those indicators and factors that have the greatest impact on the part of the power network in which the measures will be applied [30].

In [31], a model for assessing the power quality in a microgrid is proposed based on a multidimensional Gaussian distribution and local sensitivity analysis. The first is used to combine individual quality indicators and establish a correlation between them, the second is used to give the indicators different weights.

Recently, works based on artificial intelligence methods have begun to appear, and they have a bias towards the classification of power quality indicators.

Thus, a new method for classifying deviations in power quality indicators based on a deep convolutional neural network and a multiclass support vector machine is presented in [32]. S-transforms are used to obtain information about power quality factors. This option has proven to be very effective with significant data noise. Also, a convolutional neural network, but of the ensemble type, created to solve the same problem is presented in [33].

The article [34] proposes to use a deep belief network with semi-supervised learning (SDBN), and [35] – an improved densely connected network (DenseNet) using a Markov transition field for the classification of power quality indicators.

In [36], a method for classifying deviations in power quality indicators in power grids based on measured data is proposed, which is based on a random forest algorithm with weighted optimization. Here, wavelet transform, and sample entropy are used to extract information about power quality factors.

The purpose of this study is to develop a comprehensive multifactor mathematical model for assessing power quality. This approach considers multiple factors influencing power quality in underground mining environments, providing a more accurate and holistic analysis. By applying this multifactorial methodology, the study aims to improve the reliability and efficiency of power systems in mining operations.

RESEARCH METHODOLOGY AND METHODS FOR THE POWER QUALITY ANALYSIS IN UNDERGROUND MINING

This study adopts a quantitative research methodology focused on constructing and validating a multifactorial mathematical model to assess power quality in underground mining systems [37]. The approach is based on systems engineering principles and modern energy management practices [38]. Key power quality indicators, such as voltage sags, total harmonic distortion, power factor, reactive power flow, and supply continuity, were identified through a review of current literature and consultations with industry experts [37, 39]. Contextual factors specific to underground mining operations, including equipment types, load variability, network topology, and environmental conditions, were incorporated into the analysis [40].

Data collection was conducted in operational underground iron ore mines using power quality analysers, SCADA systems, and digital fault recorders to capture high-resolution electrical parameters over time [41]. Field data included voltage and current waveforms, frequency deviations, and harmonic distortions across different mining cycles. The collected data were processed and analysed using statistical tools such as principal component analysis (PCA), multiple regression, and factor analysis to establish correlations between operational and electrical variables [42]. The resulting model combines both deterministic and stochastic components, enabling the simulation of power quality behaviour under varying conditions [42, 43].

The model was implemented in MATLAB/Simulink to perform scenario-based simulations, assessing the impact of operational shifts, equipment configurations, and load dynamics on power quality [44]. Model validation was performed by comparing simulation outputs with actual measurements from the field [45]. Statistical indicators such as the coefficient of determination (R^2), root mean square error (RMSE) and mean absolute percentage error (MAPE) were used to evaluate model accuracy and reliability [46]. This multifactorial approach offers a more holistic view of power quality in underground mining and serves as a decision-support tool for improving energy efficiency, reliability, and operational safety [44, 47]

The multifactor model of power quality analysis

The multifactor model of power quality refers to the dependence F, which connects the total power quality Q with its input $\overline{q}=(q_1, q_2,\ldots,q_n)$ – the vector of power qualities of individual components:

$$Q = F(\bar{q}). \tag{1}$$

The power quality is influenced by many factors. Moreover, the influence of these factors on the power quality varies in strength. Thus, to build a mathematical multifactor model for the power quality, it is necessary to consider not only the factors themselves, but also the degree of influence of each factor on the overall power quality. According to the general approach to building a mathematical model of the object under consideration, at the first step it is necessary to carry out a structural synthesis of the model. At this stage, the type of dependence F is determined without taking into account the values of its parameters. Let us conditionally perform the following operation: "split" the model F into its structure St and parameters $c_1, c_2,\ldots, c_n$, that is, present the model in the form of a pair:

$$F = \langle St, \bar{C} \rangle, \tag{2}$$

where $\overline{C} = (c_1, c_2, \ldots, c_n)$ – vector of model parameters.

At the stage of structural synthesis, only the structure St of the model is determined, and the specific values of the parameters of the vector $\overline{C}$ are not of interest. In general, the

Tab. 1. Statistical material for building a mathematical model (36)

Tab. 1. Materiał statystyczny do budowy modelu matematycznego (36)

No	Q	q	y ln	x ln	Q
1	0.9	0.95	-0.10536	-0.05129	0.913
2	0.89	0.92	-0.11653	-0.08338	0.862
3	0.85	0.91	-0.16252	-0.09431	0.846
4	0.88	0.93	-0.12783	-0.07257	0.879
5	0.84	0.91	-0.17435	-0.09431	0.846
6	0.85	0.92	-0.16252	-0.08338	0.862
7	0.83	0.9	-0.18633	-0.10536	0.829
8	0.87	0.89	-0.13926	-0.11653	0.813
9	0.91	0.96	-0.09431	-0.04082	0.930
10	0.93	0.98	-0.07257	-0.0202	0.965

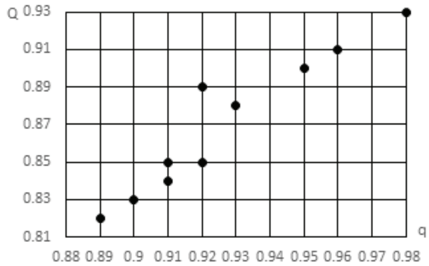

Fig. 1. Correlation field of statistical material of table 1

Rys. 1. Pole korelacji materiału statystycznego tabeli 1

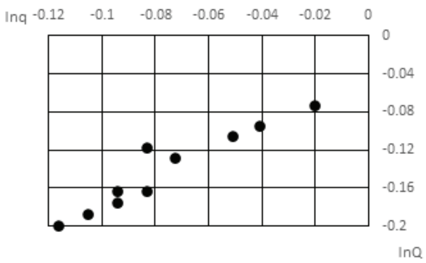

Fig. 2. Correlation field of statistical material of table 1

Rys. 2. Pole korelacji materiału statystycznego tabeli 1

structure is understood as the type of elements that make up the object and the relationships between the elements. There are many different structures of mathematical models. Linearity, stativity, determinism, discreteness, multiplicativity, etc. are structural categories.

The mathematic apparatus for model of power quality analysis

Analysis of the power quality Q as a multifactor model depending on the magnitude of the qualities of power of its components q_i, i=1, 2, …, n shows that the structure of this model can be represented as static and multiplicative.

In this case, the model is written in the form:

$$Q = k \cdot q_1^{c_1} \cdot q_2^{c_2} \cdot \ldots \cdot q_n^{c_n}, \tag{3}$$

or in a collapsed form

$$Q = k \cdot \prod_{i=1}^{n} q_i^{c_i}, \tag{4}$$

The specific values of the parameters c_1, c_2, …, c_n are not yet important, only the type of dependence is important, i.e. the multiplicative ness of the structure St.

Thus, at the stage of structural synthesis, only the type and nature of the model is determined, and its parameters are determined at the stage of identification of the model parameters. At the same time, it makes sense to analyze the properties of the selected structure of the multifactor model (3). According to the definition of power quality, the following restrictions apply:

$$0 \le Q \le 1, 0 \le q_i \le 1, (i = 1,2, \ldots, n). \tag{5}$$

Moreover, let:

$$\hat{q}_i = 1, (i = 1,2, \ldots, n), \tag{6}$$

then

$$Q = \lim_{n \to \infty} k \prod_{i=1}^{n} \hat{q}_i^{c_i} = k \lim_{n \to \infty} \prod_{i=1}^{n} \hat{q}_i^{c_i} = k \cdot 1 = k. \tag{7}$$

But under condition (6) according to (7) it holds

$$Q=1 \tag{8}$$

Considering (6) and (7), we have

$$k=1 \tag{9}$$

According to (9), the mathematical model (4) will take the form:

$$Q = \prod_{i=1}^{n} q_i^{c_i}, \tag{10}$$

Tab. 2. Results of least squares calculation
Tab. 2. Wyniki obliczeń najmniejszych kwadratów

No	y	x	x^2	$y \cdot x$
1	-0.10536	-0.05129	0.002631	0.00540429
2	-0.11653	-0.08338	0.00695249	0.00971678
3	-0.16252	-0.09431	0.0088945	0.01532727
4	-0.12783	-0.07257	0.00526651	0.00927696
5	-0.17435	-0.09431	0.0088945	0.01644339
6	-0.16252	-0.08338	0.00695249	0.01355109
7	-0.18633	-0.10536	0.01110084	0.01963178
8	-0.13926	-0.11653	0.01358013	0.02312625
9	-0.09431	-0.04082	0.00166644	0.00384995
10	-0.07257	-0.0202	0.00040815	0.00146612

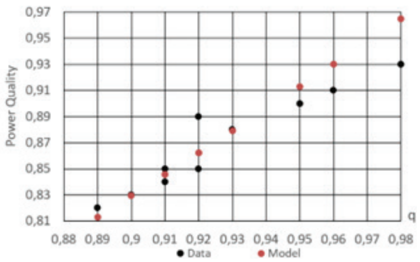

Fig. 3. Power quality graphs
Rys. 3. Wykresy jakości energii

Tab. 3. Information on the identification of the three-factor model by power quality
Tab. 3. Informacje dotyczące identyfikacji modelu trójczynnikowego według jakości energii

No	Q	q	q	q	y ln	x = ln 1	x = ln 2	x = ln 3
1	0.9	0.95	0.96	0.97	-0.105	-0.051	-0.041	-0.030
2	0.89	0.92	0.93	0.95	-0.117	-0.083	-0.073	-0.051
3	0.85	0.91	0.93	0.9	-0.163	-0.094	-0.073	-0.105
4	0.88	0.93	0.94	0.89	-0.128	-0.073	-0.062	-0.117
5	0.84	0.91	0.89	0.9	-0.174	-0.094	-0.117	-0.105
6	0.85	0.92	0.9	0.88	-0.163	-0.083	-0.105	-0.128
7	0.83	0.9	0.89	0.87	-0.186	-0.105	-0.117	-0.139
8	0.87	0.89	0.89	0.91	-0.139	-0.117	-0.117	-0.094
9	0.91	0.96	0.92	0.95	-0.094	-0.041	-0.083	-0.051
10	0.93	0.98	0.95	0.97	-0.073	-0.020	-0.051	-0.030

The obtained structure of the model (10) meets all the requirements of (5).

Let us define the content of the parameters c_1, c_2, …, c_n in the model (10). If the parameter c_i=0, then the component q_i which determines the local power quality, is not taken into account.

To find out the content of the parameters:

$$\ln Q = \ln\left(\prod_{i=1}^{n} q_i^{c_i}\right), \ln Q = \sum_{i=1}^{n} c_i \cdot \ln q_i . \tag{11}$$

In formula (11), we calculate the partial derivative with respect to the variable q_i:

$$\frac{\partial \ln Q}{\partial q_i} = \frac{\partial}{\partial q_i}\left(\sum_{i=1}^{n} c_i \cdot \ln q_i\right) = \frac{c_i}{q_i}, \frac{1}{Q}\frac{\partial Q}{\partial q_i} = \frac{c_i}{q_i}. \tag{12}$$

Thus, according to (12), the value of the parameter c_i determines the sensitivity of the overall quality Q relative to the quality component q_i, that is, how strongly the value of the quality component q_i affects the value of the overall quality Q. It is clear that the following constraint must also be satisfied:

$$c_i \geq 0, (i = 1,2, \ldots, n). \tag{13}$$

The next stage of model synthesis is to identify the parameters of the model c_i, which is associated with the determination of the numerical values of the parameters $\overline{C}=(c_1,c_2,\ldots,c_n)$. The initial information for identification is the structure St and observation of the behavior of the input $\overline{q}(t)$ and output Q(t) of the object in real conditions. Thus, the pair

$$I(t) = \langle \bar{q}(t); Q(t) \rangle, \tag{14}$$

in general, it is the main source of information for identification.

Considering that observations of the input states $\overline{q}(t)$ and output Q(t) during operation are carried out at discrete moments of time, formula (14) takes the form:

$$I = \langle \bar{q}_k; Q_k \rangle, k = 1,2, \ldots, N, \tag{15}$$

where k – the number of time points t_k, when the values $\overline{q}(t)$ та Q(t), were recorded, i.e. $q_k=\overline{q}(t_k)$,$Q_k=Q(t_k)$.

Thus, the initial data required for identification is formed by two:

$$\langle St, I \rangle, \tag{16}$$

that is, the structure of the model (10) and the observations (15).

The process of determining the parameters of the model (10) is reduced to determining the parameters $\overline{C}=(c_1, c_2, \ldots, c_n)$ from the initial data (16), i.e.:

$$C = \varphi(St, I), \tag{17}$$

where φ – an identification algorithm that determines how to find the parameters $\overline{C}=(c_1, c_2, \ldots, c_n)$, knowing St and I.

In practice, two types of identification algorithms are used: adaptive and non-adaptive. An adaptive identification algorithm is an algorithm that allows you to refine the values of the model parameters being identified as additional information is obtained. In contrast to the adaptive identification algorithm, a non-adaptive algorithm allows you to obtain the desired parameters $\overline{C}=(c_1, c_2, \ldots, c_n)$ at once, using all the information (15), and not by gradually refining them.

Given that the static model (10) is considered, it is advisable to use a non-adaptive identification algorithm to find the values of the parameters. In this case, the initial data take the form (15). According to the non-adaptive identification algorithm, it is necessary to substitute data (15) into formula (10). As a result, we obtain a system of N equations with n unknowns

$$\begin{aligned} &q_{1,1}^{c_1} \cdot q_{2,1}^{c_2} \cdot \ldots \cdot q_{n,1}^{c_n} = Q_1; \\ &q_{1,2}^{c_1} \cdot q_{2,2}^{c_2} \cdot \ldots \cdot q_{n,2}^{c_n} = Q_2; \\ &\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ &q_{1,N}^{c_1} \cdot q_{2,N}^{c_2} \cdot \ldots \cdot q_{n,N}^{c_n} = Q_N, \end{aligned}$$

which is conveniently written in the form

$$\prod_{i=1}^{n} q_{i,k}^{c_i} = Q_k, k = 1,2, \ldots, N. \tag{18}$$

The condition N≥n. must be fulfilled. Otherwise, the number of unknowns will be less than the number of equations and system (18) will not have a unique solution.

Thus, the problem of non-adaptive identification of the static model (10) is reduced to solving the system of equations (18). This system has two essential properties that determine the difficulties that arise when solving it: incompatibility and transcendence. Its incompatibility is associated with the case N>n (the number of equations is greater than the unknowns), and transcendence is associated with an arbitrary choice of the model structure. It is obvious that the problems of the non-adaptive method of solving the identification problem are associated with solving an incompatible transcendental system of equations. Let's start with incompatibility, that is, when N>n.

We solve the inconsistent system of equations (18) by the method of least squares, i.e., by minimizing the total residual of the right and left sides of the equations of this system. To do this, we form the total residual function in the form of the sum of the squares of the residuals of each of the equations:

$$F(c_1, c_2, \ldots, c_n) = \sum_{k=1}^{N} \left[\prod_{i=1}^{n} q_{i,k}^{c_i} - Q_k \right]. \tag{19}$$

Function (19) is non-negative and equals zero when the right and left sides of the equations of system (18) coincide. In this case, the closer the right sides of the equation of system (18) are to the left sides, the smaller the value of the residual function (19). This gives grounds to consider the following values of the parameters as solutions of system (18)

$$\bar{C}^* = (c_1^*, c_2^*, \ldots c_n^*), \tag{20}$$

at which the residual function (19) is minimal, i.e.:

$$F(c_1^*, c_2^*, \ldots c_n^*) = \min_{c_1, c_2, \ldots, c_n} F(c_1, c_2, \ldots, c_n).$$

Therefore, to solve the incompatible system of equations (18), it is enough to minimize the total residual function (19).

RESULTS OF THE POWER QUALITY ANALYSIS FOR UNDERGROUND MINING

The results of our research are based on solving the outlined tasks, which primarily involved the development of a mathematical framework and the implementation of targeted investigations. These studies focused on establishing key parameters for power quality analysis in underground mining environments. The applied methodology ensured a systematic approach to analyzing energy quality indicators under complex geological and operational conditions. By integrating analytical modeling with empirical data, we achieved a more accurate interpretation of power disturbances and their sources. This comprehensive approach lays the foundation for future optimization of energy systems in underground mining infrastructure.

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For the identification of the function (18), the total residual function (19) is quite complicated, since it includes exponential functions. Therefore, it is proposed to apply such a transformation that would convert the multiplication operation into the addition operation without changing the result of minimizing the function (19). For this, it is possible to apply the action of logarithmization, which converts multiplication into addition. As a result, formula (10) will take the form (11):

$$\ln Q = \sum_{i=1}^{n} c_i \cdot \ln q_i.$$

For the convenience of further calculations, it is advisable to introduce the notation:

$$\ln Q = y, \ln q_i = x_i. \tag{21}$$

Then formula (11) taking into account (21) will take the form:

$$y = \sum_{i=1}^{n} c_i \cdot x_i \,. \tag{22}$$

The specificity of such a structure lies in the linearity of the function with respect to the parameters being sought, c_1, c_2, …, c_n.

In turn, the total residual function will be written as

$$S(c_1, c_2, \ldots, c_n) = \sum_{k=1}^{N} \left(\sum_{i=1}^{n} c_i \cdot x_{i,k} - y_k \right)^2. \tag{23}$$

Tab. 4. Results of calculations of coefficients of the system of equations (40)

Tab. 4. Wyniki obliczeń współczynników układu równań (40)

No	x^2	$x\ x$	$x\ x$	x^2	$x\ x$	x^2	$x\ y$	$x\ y$	$x\ y$
1	0.002631	0.002094	0.001562	0.001666	0.001243	0.000928	0.005404	0.004301	0.003209
2	0.006952	0.006051	0.004277	0.005267	0.003722	0.002631	0.009717	0.008457	0.005977
3	0.008895	0.006844	0.009937	0.005267	0.007646	0.011101	0.015327	0.011794	0.017123
4	0.005267	0.00449	0.008457	0.003829	0.007211	0.01358	0.009277	0.00791	0.014897
5	0.008895	0.01099	0.009937	0.01358	0.012278	0.011101	0.016443	0.020318	0.01837
6	0.006952	0.008785	0.010659	0.011101	0.013469	0.016341	0.013551	0.017123	0.020775
7	0.011101	0.012278	0.014673	0.01358	0.016229	0.019394	0.019632	0.021714	0.025949
8	0.01358	0.01358	0.01099	0.01358	0.01099	0.008895	0.016229	0.016229	0.013134
9	0.001666	0.003404	0.002094	0.006952	0.004277	0.002631	0.00385	0.007864	0.004838
10	0.000408	0.001036	0.000615	0.002631	0.001562	0.000928	0.001466	0.003722	0.00221
Σ	0.066347	0.069553	0.073201	0.077453	0.078628	0.087529	0.110896	0.119432	0.126482

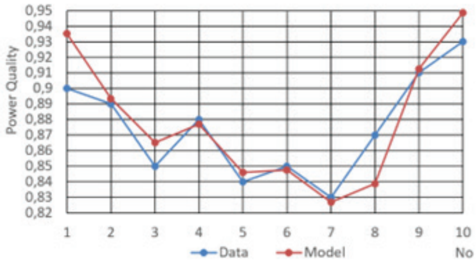

Fig. 4. Graphs of real power quality values and those calculated by the model (54)

Rys. 4. Wykresy wartości jakości energii elektrycznej rzeczywistej i obliczonej przez model (54)

This linearity allows us to reduce the problem of minimizing the residual function (23) to solving a system of linear algebra equations.

The simple form of the function (22) allows us to solve the problem of minimizing (23) by equating the partial derivatives of the function (23) to zero, in accordance with the necessary condition for the existence of an extremum, i.e.

$$\frac{\partial}{\partial c_j} S(c_1, c_2, \dots, c_n) = \frac{\partial}{\partial c_j}\left[\sum_{k=1}^{N}\left(\sum_{i=1}^{n} c_i \cdot x_{i,k} - y_k\right)^2\right] = 0, \tag{24}$$

$$j = 1,2, \dots, n.$$

Since function (22) is a linear function with respect to the parameters c_1, c_2, …, c_n, then $S(c_1, c_2, \dots, c_n)$ in (23) is a quadratic function, which determines the linearity of the system of equations (24). Indeed, calculating the partial derivative in (24), we have

$$\frac{\partial}{\partial c_j}\left[\sum_{k=1}^{N}\left(\sum_{i=1}^{n} c_i \cdot x_{i,k} - y_k\right)^2\right] =$$
$$= 2\sum_{k=1}^{N}\left(\sum_{i=1}^{n} c_i \cdot x_{i,k} - y_k\right) x_{j,k} = 0,$$
$$\sum_{k=1}^{N}\left(\sum_{i=1}^{n} c_i \cdot x_{i,k} - y_k\right) x_{j,k} = \sum_{i=1}^{n} c_i \cdot \varphi_{i,j} - \eta_j = 0,$$

де $\varphi_{i,j} = \sum_{k=1}^{N} x_{i,k} \cdot x_{j,k}$; $\eta_j = \sum_{k=1}^{N} y_k \cdot x_{j,k}$, $j, i = 1,2, \dots, n$.

As can be seen, (25) is a system of linear algebra equations with respect to the identified parameter c_1, c_2, …, c_n,

$$\begin{cases} c_1 \cdot \varphi_{1,1} + c_2 \cdot \varphi_{2,1} + \dots + c_n \cdot \varphi_{n,1} = \eta_1; \\ c_1 \cdot \varphi_{1,2} + c_2 \cdot \varphi_{2,2} + \dots + c_n \cdot \varphi_{n,2} = \eta_2; \\ \vdots \\ c_1 \cdot \varphi_{1,n} + c_2 \cdot \varphi_{2,n} + \dots + c_n \cdot \varphi_{n,n} = \eta_n, \end{cases} \tag{26}$$

which is solved by standard methods.

To study the solution of the system of equations (26), we consider the matrix of this system of coefficients with unknown parameters c_1, c_2, …, c_n of size (n×n):

$$\Phi = \begin{bmatrix} \varphi_{1,1} & \varphi_{2,1} & \cdots & \varphi_{n,1} \\ \varphi_{1,2} & \varphi_{2,2} & \cdots & \varphi_{n,2} \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_{1,n} & \varphi_{2,n} & \cdots & \varphi_{n,n} \end{bmatrix}, \tag{27}$$

This matrix is called the Fisher information matrix. It is symmetric, since $\varphi_{i,j}=\varphi_{j,i}$. For a unique solution of system (26) it is necessary that the determinant of this matrix is not equal to zero, i.e.

$$|\Phi| = \begin{bmatrix} \varphi_{1,1} & \varphi_{2,1} & \cdots & \varphi_{n,1} \\ \varphi_{1,2} & \varphi_{2,2} & \cdots & \varphi_{n,2} \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_{1,n} & \varphi_{2,n} & \cdots & \varphi_{n,n} \end{bmatrix} \neq 0. \tag{28}$$

This determinant is zero in two cases: when, that is, when there are not enough measurements, and when there is not enough variability x_1, x_2, …, x_n, that is, when more than N-n states are linearly dependent. There are two ways to respond to this situation: either increase the number of observations N, or decrease the number n. There are two ways to respond to this situation: either increase the number of observations N, or decrease the number n.

Let us consider Cramer's method for solving the system of algebraic equations (26).

For this, Cramer's formulas are used. First, the determinant according to the system of equations (26) is composed of coefficients with unknown parameters c_1, c_2, …, c_n:

$$\Delta = \begin{bmatrix} \varphi_{1,1} & \varphi_{2,1} & \cdots & \varphi_{n,1} \\ \varphi_{1,2} & \varphi_{2,2} & \cdots & \varphi_{n,2} \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_{1,n} & \varphi_{2,n} & \cdots & \varphi_{n,n} \end{bmatrix}. \tag{29}$$

If $\Delta \neq 0$, then we compose the determinants, successively replacing the columns in (29) with the columns of the free terms of the system (26)

$$\Delta_1=\begin{bmatrix}\eta_1 & \varphi_{2,1} & \cdots & \varphi_{n,1}\\ \eta_2 & \varphi_{2,2} & \cdots & \varphi_{n,2}\\ \cdots & \cdots & \cdots & \cdots\\ \eta_n & \varphi_{2,n} & \cdots & \varphi_{n,n}\end{bmatrix},\Delta_2=\begin{bmatrix}\varphi_{1,1} & \eta_1 & \cdots & \varphi_{n,1}\\ \varphi_{1,2} & \eta_2 & \cdots & \varphi_{n,2}\\ \cdots & \cdots & \cdots & \cdots\\ \varphi_{1,n} & \eta_n & \cdots & \varphi_{n,n}\end{bmatrix},\ldots,$$
$$\Delta_\mathrm{n}=\begin{bmatrix}\varphi_{1,1} & \varphi_{2,1} & \cdots & \eta_1\\ \varphi_{1,2} & \varphi_{2,2} & \cdots & \eta_2\\ \cdots & \cdots & \cdots & \cdots\\ \varphi_{1,n} & \varphi_{2,n} & \cdots & \eta_n\end{bmatrix}. \tag{30}$$

The solution of system (26) is found by Cramer's formula by successively dividing (30) by (29):

$$c_1^\times=\frac{\Delta_1}{\Delta};\ c_2^\times=\frac{\Delta_2}{\Delta},\ldots,c_n^\times=\frac{\Delta_n}{\Delta}. \tag{31}$$

Thus, the parameter values (31) minimize the residual function (23), i.e.

$$S(c_1^\times,c_2^\times,\ldots,c_n^\times)=\min_{c_1,c_2,\ldots,c_n}S(c_1,c_2,\ldots,c_n).$$

Moreover, these parameter values (30) coincide with parameters (20), i.e.

Minimize and function (19), i.e. executes

$$F(c_1^\times,c_2^\times,\ldots,c_n^\times)=\min_{c_1,c_2,\ldots,c_n}F(c_1,c_2,\ldots,c_n).$$

The multifactor mathematical model for power quality according to (10) and (31) will take the form:

$$Q_m=\prod_{i=1}^{n}q_i^{c_i^\times}, \tag{32}$$

Model (32) allows us to estimate the relative error in determining the power quality based on the known errors in the qualities of its components. Indeed, let us take the logarithm of (32) and calculate the differential:

$$\begin{gathered}\ln Q_m=\sum_{i=1}^{n}c_i^\times\cdot\ln q_i;\\ d(\ln Q_m)=d\left(\sum_{i=1}^{n}c_i^\times\cdot\ln q_i\right),\\ \frac{dQ_m}{Q_m}=\sum_{i=1}^{n}c_i^\times\cdot\frac{dq_i}{q_i}.\end{gathered} \tag{33}$$

Assuming that

$$dQ_m\approx\Delta Q_m, dq_i\approx\Delta q_i, i=1,2,\ldots,n, \tag{34}$$

we obtain the desired formula:

$$\delta Q_m=\sum_{i=1}^{n}c_i^\times\cdot\delta q_i, \tag{35}$$

where $\delta Q_m=\frac{\Delta Q_m}{Q_m};\delta q_i=\frac{\Delta q_i}{q_i}, i=1,2,\ldots,n$ – relative errors of power quality.

Let us consider the construction of a multifactor model for the power quality using an example.

It is advisable, for the purpose of a consistent in-depth study, to consider a single-factor model for the power quality using an example, the general form of which is as follows

$$Q=f(q) \tag{36}$$

where Q – power quality,
q – partial power quality.

According to formula (36), the input variable is the partial power quality q, and the output variable is the power quality Q.

To build a model that corresponds to formula (36), we will use the statistical material presented in table 1.

Fig. 1 shows the correlation field (q,Q) of the statistical material presented in Table 1.

The analysis of the correlation field presented in Fig. 1 shows that its character corresponds to all its features, which were considered above in the general case. This allows us to choose the structure of formula (36) in the form of a power function

$$Q=q^c, \tag{37}$$

where c – parameter.

Analysis of formula (37) shows that to find its parameter it is convenient to use the logarithm operation, which will bring it to a linear form

$$\ln Q=c\ln q,$$

or

$$y=c\cdot x, \tag{38}$$

where y=ln Q, x=ln q.

Fig. 2 presents the correlation field (ln q, ln Q) of the statistical material in Table 1.

Analysis of the correlation field in Fig. 2 shows that there is a linear relationship between the variables, which confirms the entry of formula (38).

To find the value of the parameter c, we use the method of least squares. Since there are more equations than parameters (10 >1), we construct the total residual function

$$F(c)=\sum_{i=1}^{10}(cx_i-y_i)^2. \tag{39}$$

To find the parameter c, we minimize the function (39) with respect to the desired parameter. We use the necessary condition for the existence of an extremum of the function, according to which the derivative of the function at the extremum point is zero

$$\begin{gathered}F'(c)=\left[\sum_{i=1}^{10}(cx_i-y_i)^2\right]'=0;\\ \left[\sum_{i=1}^{10}(cx_i-y_i)^2\right]'=2\sum_{i=1}^{10}(cx_i-y_i)x_i=0;\\ \sum_{i=1}^{10}(cx_i-y_i)x_i=\sum_{i=1}^{10}(cx_i^2-y_ix_i)=0;\\ \sum_{i=1}^{10}(cx_i^2-y_ix_i)=c\sum_{i=1}^{10}x_i^2-\sum_{i=1}^{10}y_ix_i=0;\\ c\sum_{i=1}^{10}x_i^2=\sum_{i=1}^{10}y_ix_i;\\ c=\frac{\sum_{i=1}^{10}y_ix_i}{\sum_{i=1}^{10}x_i^2}.\end{gathered} \tag{40}$$

According to the data in Table 2, using formula (40), we calculate

$$c=\frac{\sum_{i=1}^{10}y_ix_i}{\sum_{i=1}^{10}x_i^2}=\frac{0.11779387}{0.06634705}=1.775. \tag{41}$$

Taking into account (37), the single-factor model for power quality according to (41) has the form:

$$Q=q^{1.775}. \tag{42}$$

Fig. 3 shows graphs of power quality based on statistical data, according to Table 1, and calculations based on model (42).

Analysis of the graphs shown in Fig. 3 shows a fairly good match. The correlation coefficient was r=0.951. (43)

The assessment of the closeness of the relationship between variables from the perspective of the Chaddock scale [48] is a "very high" value, since, according to (43),

$$0.9 \le r \le 0.99. \quad (44)$$

Next, we will consider a more complex example, namely, the construction of a three-factor model for the power quality on the corresponding example.

Let three parameters of the power quality be selected: voltage U, frequency f, shape of the electric current curve . It is known that the quality of voltage U is defined as q_1, the quality of the frequency f-q_2 and the quality of the shape of the electric current curve χ-q_3. Then the multifactor model for the power quality will be written in the form:

$$Q = q_1^{c_1} \cdot q_2^{c_2} \cdot q_3^{c_3}. \quad (45)$$

To identify the model (45), i.e. to find the unknown parameters $c_1, c_2, \ldots, c_3$, it is necessary to have statistical material regarding the qualities included in the model (45). The corresponding statistical material is presented in Table 3. According to the data in Table 3, the volume of statistics was 10 data. The first four columns contain the initial information regarding the general power quality and the qualities of power of three parameters: voltage, frequency and shape of the electric current curve. The following four columns present the results of logarithmic transformations of the initial data. As a result of logarithmic transformations of the initial data, the mathematical model took the form:

$$y = c_1 x_1 + c_2 x_2 + c_3 x_3. \quad (46)$$

In this case, the total residual function will be written as:

$$S(c_1, c_2, c_3) = \sum_{k=1}^{10} (c_1 x_1 + c_2 x_2 + c_3 x_3 - y_k)^2. \quad (47)$$

According to the necessary minimization condition (47), we set the partial derivative with respect to the unknown parameter c_i to zero:

$$\frac{\partial}{\partial c_i} S(c_1, c_2, c_3) = = 2\sum_{k=1}^{10} (c_1 x_{1,k} + c_2 x_{2,k} + c_3 x_{3,k} - y_k) x_{i,k} = 0, \quad i = 1,2,3. \quad (48)$$

Expanding condition (48), we obtain a system of three linear algebraic equations with three unknowns $c_1, c_2, \ldots, c_3$:

$$\begin{cases} c_1 \cdot \sum_{k=1}^{10} x_{1,k}^2 + c_2 \cdot \sum_{k=1}^{10} x_{1,k} x_{2,k} + c_3 \cdot \sum_{k=1}^{10} x_{1,k} x_{3,k} = \\ = \sum_{k=1}^{10} x_{1,k} y_k \\ c_1 \cdot \sum_{k=1}^{10} x_{2,k} x_{1,k} + c_2 \cdot \sum_{k=1}^{10} x_{2,k}^2 + c_3 \cdot \sum_{k=1}^{10} x_{2,k} x_{3,k} = \\ = \sum_{k=1}^{10} x_{2,k} y_k \\ c_1 \cdot \sum_{k=1}^{10} x_{3,k} x_{1,k} + c_2 \cdot \sum_{k=1}^{10} x_{3,k} x_{2,k} + c_3 \cdot \sum_{k=1}^{10} x_{3,k}^2 = \\ = \sum_{k=1}^{10} x_{3,k} y_k \end{cases} \quad (49)$$

The results of calculating the sums present in the system of equations (49) are presented in Table 4.

According to the data in Table 4, the system of equations (49) takes the form:

$$\begin{cases} 0.066347c_1 + 0.069553c_2 + 0.073201c_3 = 0.11089 \\ 0.069553c_1 + 0.077453c_2 + 0.078628c_3 = 0.11943 \\ 0.073201c_1 + 0.078628c_2 + 0.0875229c_3 = 0.1264 \end{cases} \quad (50)$$

The Fisher matrix of system (41) has the form:

$$\Phi = \begin{bmatrix} 0.066347 & 0.069553 & 0.073201 \\ 0.069553 & 0.077453 & 0.078628 \\ 0.073201 & 0.078628 & 0.087529 \end{bmatrix}. \quad (51)$$

Since the determinant of the Fisher matrix (51) is not zero:

$$|\Phi| = 1.8 \cdot 10^{-6} \neq 0,$$

then the system of equations has a unique solution.

Let's find this solution using Cramer's rules. We find the determinants:

$$\Delta = \begin{vmatrix} 0.066347 & 0.069553 & 0.073201 \\ 0.069553 & 0.077453 & 0.078628 \\ 0.073201 & 0.078628 & 0.087529 \end{vmatrix} = 1.80065 \cdot 10^{-6},$$

$$\Delta_1 = \begin{vmatrix} 0.110896 & 0.069553 & 0.073201 \\ 0.119432 & 0.077453 & 0.078628 \\ 0.126482 & 0.078628 & 0.087529 \end{vmatrix} = 1.12476 \cdot 10^{-6},$$

$$\Delta_2 = \begin{vmatrix} 0.066347 & 0.110896 & 0.073201 \\ 0.069553 & 0.119432 & 0.078628 \\ 0.073201 & 0.126482 & 0.087529 \end{vmatrix} = 9.08189 \cdot 10^{-7},$$

$$\Delta_3 = \begin{vmatrix} 0.066347 & 0.069553 & 0.110896 \\ 0.069553 & 0.077453 & 0.119432 \\ 0.073201 & 0.078628 & 0.126482 \end{vmatrix} = 8.45526 \cdot 10^{-7}.$$

Then:

$$c_1 = \frac{\Delta_1}{\Delta} = 0.624641, c_2 = \frac{\Delta_2}{\Delta} = 0.504369, \quad c_3 = \frac{\Delta_3}{\Delta} = 0.469568. \quad (52)$$

Thus, taking into account (52), formula (46) takes the form:

$$y_m = 0.624641x_1 + 0.504369x_2 + 0.469568x_3. \quad (53)$$

Taking into account the use of logarithms in calculations, formula (45) can be written as follows:

$$Q_m = q_1^{0.624641} \cdot q_2^{0.504369} \cdot q_3^{0.469568}. \quad (54)$$

Fig. 4 presents graphs of real values of power quality and calculated by the mathematical model (45).

Comparison of the graphs shown in Fig. 4 shows their fairly good similarity. Moreover, the calculated pair correlation coefficient is:

$$r_{QQm} = 0.923. \quad (55)$$

According to the Chaddock scale [48], since, according to (55), the inequality holds:

$$0.9 < r_{QQm} < 0.99,$$

then there is a "very high" relationship between the variables.

Thus, formula (45) represents a three-factor mathematical model for the power quality.

Using formula (35), it is possible to write the relative error of the overall power quality depending on the relative errors of the qualities of power of the three-factor mathematical model for the power quality:

$$\delta Q_m = 0.624641\delta q_1 + 0.504369\delta q_2 + 0.469568\delta q_3,$$

where δq_1 – relative voltage quality error U,
δq_2 – relative frequency quality error ω,
δq_3 – relative error in the quality of the shape of the electric current curve χ.

DISCUSSION OF RESEARCH RESULTS

The identification of the functional dependence of power quality Q on a set of partial quality factors qi has been approached through a log-linear transformation to simplify the minimization of a nonlinear residual function. Originally expressed in a multiplicative form involving exponentials (equation 10), the model posed challenges for direct optimization. By applying a logarithmic transformation, the model was linearized (equation 11), enabling the reduction of the problem to a classical linear regression framework.

This transformation brings significant analytical advantages. First, it converts the product of power functions into a summation, preserving the fundamental structure of the model while aligning it with least-squares estimation techniques. The transformed model (equation 22) is linear with respect to the unknown parameters ci, making the minimization of the residual function (equation 23) tractable through standard linear algebra methods.

The derivation of the normal equations (equation 24) leads to a system of linear equations (equation 26), solvable either via matrix inversion or Cramer's rule. The Fisher Information Matrix Φ (equation 27) plays a central role in assessing the identifiability and uniqueness of the solution. It is emphasized that for the system to yield a unique solution, the determinant of this matrix must be nonzero (equation 28), which hinges on the number of independent observations and the linear independence of the input data.

In case of rank deficiency (i.e., when $|\Phi|=0$), two practical remedies are proposed: increase the number of observations N or reduce the number of parameters n. The application of Cramer's rule (equations 29–31) not only provides a direct computational path for determining the parameter estimates ci× but also reinforces the linkage between these estimates and the minimization of both the residual function S and the original total residual function F.

The derived multifactor model (equation 32) provides a robust framework for estimating the integrated quality index Q_m. Further, the analytical derivation of the model's sensitivity to input uncertainty (equations 33–35) is of substantial practical importance. The model quantifies how the relative errors in partial indicators δ_{qi} propagate into the overall quality estimate δQm, which is vital for risk assessment and reliability analysis.

To validate the methodology, a simplified single-factor case (equation 36) was investigated using empirical data (Table 1). Logarithmic transformation again enabled a linearized form (equation 38), with the parameter c estimated via the least squares criterion (equations 39–40). Numerical results (Table 2) yielded a value of c = 1.775 (equation 41), leading to a fitted model $Q = q^{1.775}$ (equation 42). The graphical comparison (Figure 3) and the computed correlation coefficient r = 0.951 (equation 43) demonstrate an excellent agreement between the model and the observed data, falling within the "very high" correlation range on the Chaddock scale (equation 44).

This single-factor model illustrates the effectiveness of the proposed methodology and serves as a foundation for constructing more complex multifactor models. By extending this technique to include additional variables, the robustness and predictive capabilities of the model can be enhanced, which will be explored in subsequent sections.

CONCLUSION

In this study, a rigorous mathematical framework was developed for identifying parameters within a multifactor power quality model. The proposed approach is grounded in the logarithmic transformation of a multiplicative function, which linearizes the structure and facilitates the use of the least squares method. This transformation allowed for the derivation of a total residual function in a quadratic form, enabling analytical minimization through partial derivatives and solution of a system of linear algebraic equations. The use of Cramer's rule further demonstrated the analytical solvability of the system, provided that the determinant of the Fisher information matrix is non-zero, ensuring a unique solution.

In the conditions of market relations, it is advisable to consider power as a commodity that must meet a certain quality. This requires paying special attention to the power quality, which in turn is determined by the indicators of the components that make it up. Building a multifactor mathematical model of power quality in the form of a static multiplicative model makes it possible to study the influence of the components of individual component indicators of power on the overall quality, and, ultimately, to highlight ways to improve the power quality, relying on the quality indicators of the components that form the overall power quality. In addition, the synthesized multifactor mathematical model allows us to estimate the relative error in determining the overall power quality as a weighted sum of the relative errors of the power quality indicators of its components.

A single-factor power quality model was constructed and validated using empirical data, confirming the theoretical basis of the proposed method. By the power function $Q = q^c$, the model was successfully linearized and calibrated with statistical data. The parameter c was estimated via the least squares method, resulting in a highly accurate model with a correlation coefficient of r =0.951, which, according to Chaddock's scale, indicates a very strong correlation. This example not only validates the efficacy of the proposed method but also highlights its practical applicability in real-world scenarios where component-wise power quality assessment is required.

The findings of this research form a reliable foundation for extending the methodology to multifactor models of higher complexity. The generalized approach enables modelling scenarios involving multiple quality-determining variables, maintaining computational tractability and analytical transparency. Furthermore, the derived formula for relative error propagation offers a valuable tool for evaluating the robustness of the model under uncertain input conditions. As a result, the proposed mathematical model serves as both a practical and theoretically sound instrument for analysing and optimizing power quality in multifactorial systems.

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Wieloczynnikowe podejście do analizy jakości energii w górnictwie podziemnym

Niniejsze badania przedstawiają opracowanie wieloczynnikowego statycznego modelu multiplikatywnego do analizy jakości energii elektrycznej w podziemnych systemach energetycznych górnictwa. Celem jest synteza uogólnionego wskaźnika jakości energii elektrycznej poprzez integrację kluczowych parametrów, takich jak spadki i zapady napięcia, odchylenia częstotliwości, zniekształcenia harmoniczne i inne krytyczne wskaźniki wpływające na efektywność energetyczną i niezawodność sieci elektrycznej. Proponowaną strukturę modelu opracowano z wykorzystaniem metody syntezy, a jej parametry zidentyfikowano za pomocą podejścia maladaptacyjnego opartego na metodzie najmniejszych kwadratów. Aby zweryfikować dokładność modelu, wykorzystano techniki statystyki matematycznej. W rezultacie wyprowadzono zależności matematyczne do oceny uogólnionego wskaźnika jakości energii elektrycznej, wykorzystując dane dotyczące spadku napięcia, odchylenia częstotliwości i zniekształceń harmonicznych. Model, scharakteryzowany jako statyczny i multiplikatywny, wymaga pełnego spektrum danych jakościowych do identyfikacji parametrów za pomocą podejścia nieadaptacyjnego. Porównawcza analiza dokładności między modelem jednoczynnikowym a proponowanym modelem trójczynnikowym wykazała współczynnik korelacji wynoszący 0,951 dla pierwszego i 0,923 dla drugiego. Chociaż model wieloczynnikowy wykazuje spadek dokładności statystycznej o 2,94%, oba modele charakteryzują się „bardzo wysoką" niezawodnością według skali Chaddocka. Potwierdza to praktyczną przydatność podejścia wieloczynnikowego w rzeczywistych systemach energetycznych górnictwa. Nowość naukowa tkwi w ulepszonej strukturze modelu wieloczynnikowego, która syntetyzuje wiele wskaźników jakości w ujednolicone ramy. Jego praktyczna wartość jest widoczna w zastosowaniach do zarządzania przepływem energii w przemysłowych mikrosieciach w kopalniach podziemnych, zwłaszcza tych integrujących lokalne źródła energii.

Słowa kluczowe: *jakość energii, górnictwo podziemne, model wieloczynnikowy, spadki napięcia, efektywność energetyczna, statyczny model multiplikatywny, identyfikacja maladaptacyjna, systemy energetyczne kopalni, metoda najmniejszych kwadratów, zarządzanie przepływem energii*